A Theory of Tacit Collusion*

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Abstract

A theory of tacit collusion is developed based on coordination through price leadership and less than full mutual understanding of strategies. It is common knowledge that price increases are to be at least matched but who should lead and at what price is not common knowledge. The steady-state price is characterized and it falls short of the best collusive equilibrium price. Coordination through tacit means, rather than express communication, is then shown to constrain the extent of the price rise from collusion.

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1 Introduction

The economic theory of collusion focuses on what outcomes are sustainable and the strategy profiles that sustain them: What prices and market allocations can be supported? What are the most effective strategies for monitoring compliance? What are the most severe punishments that can be imposed in response to evidence of non-compliance? The literature is rich in taking account of the determinants of the set of collusive outcomes including market traits such as product differentiation and demand volatility, firm traits such as capacity, cost, and time preference, and the amount of public and private information available to firms.

In comparison, the primary focus of antitrust law is not on the outcome nor the strategies that sustain an outcome but rather the means by which a collusive arrangement is achieved. The illegality comes from firms having an agreement to coordinate their behavior.

> [A]ntitrust law clarified that the idea of an agreement describes a process that firms engage in, not merely the outcome that they reach. Not every parallel pricing outcome constitutes an agreement because not every such outcome was reached through the process to which the law objects: a negotiation that concludes when the firms convey mutual assurances that the understanding they reached will be carried out.[1]

To establish the presence of an agreement - and thereby a violation of Section 1 of the Sherman Act - it must be shown that firms "had a conscious commitment to a common scheme designed to achieve an unlawful objective,"[2] that they had a "unity of purpose or a common design and understanding, or a meeting of minds."[3] Thus, the law focuses on what mutual understanding exists among firms and how that mutual understanding was achieved.[4]

From this perspective, U.S. antitrust law has identified three types of collusion. *Explicit collusion* is when supracompetitive prices are achieved via express communication about an agreement; there has been a direct exchange of assurances regarding the coordination of their conduct. Mutual understanding is significant and is acquired through express communication. Explicit collusion is illegal. *Conscious parallelism* is when supracompetitive prices are achieved without express communication. A common example is two adjacent gasoline stations in which one station raises its price to a supracompetitive level and the other station matches the price hike. While there may be mutual understanding regarding the underlying mechanism that stabilizes those supracompetitive prices (for example, any price undercutting results in a return to

[1] Baker (1993), p. 179.

[2] *Monsanto Co. v. Spray-Rite Serv. Corp.*, 465 U.S. 752 (1984); 753.

[3] *American Tobacco Co. v. United States*, 328 U.S. 781 (1946); 810.

[4] The distinction between the economic and legal approaches to collusion is presented in Kaplow and Shapiro (2007); also, see Kaplow (2011a, 2011b, 2011c).

competitive prices), this understanding was not reached through express communication. Conscious parallelism is legal.[5] *Concerted action* resides between these two extremes and refers to when supracompetitive prices are achieved with some form of direct communication - such as about intentions - but firms do not expressly propose and reach an agreement (Page 2007).[6] For example, concerted practices may involve a firm's public announcement of a proposed pricing policy which, without the express affirmative response from its rivals, is followed by the common adoption of that policy with a subsequent rise in price. The extent of mutual understanding is more than conscious parallelism but does not reach the level of explicit collusion. Concerted action lies in the gray area of what is legal and what is not. Conscious parallelism and concerted action are both forms of tacit collusion in that a substantive part of the collusive arrangement is achieved without express communication.

While the distinction between explicit and tacit collusion exists in practice and in the law, it is a distinction that is largely absent from economic theory.[7] The economic theory of collusion - based on equilibrium analysis - presumes mutual understanding is complete (that is, the strategy profile is common knowledge) and does not deal with how mutual understanding is achieved, nor the extent of coordinated behavior that can result when there are gaps in mutual understanding. Furthermore, there is good reason for firms to try to collude without express communication, and thus find themselves dealing with less than full mutual understanding. Given that explicit collusion is illegal and tacit collusion often escapes conviction, if firms can achieve a collusive outcome through tacit means then they will presumably do so and thereby avoid the possibility of financial penalties and jail time. This then leads one to ask: What types of markets are conducive to tacit collusion? What types of public announcements are able to generate sufficient mutual understanding to produce collusion? In markets for

[5]Conscious parallelism "refers to a form of tacit collusion in which each firm in an oligopoly realizes that it is within the interests of the entire group of firms to maintain a high price or to avoid vigorous price competition, and the firms act in accordance with this realization." (Hylton, 2003, p. 73)

[6]From *Interstate Circuit, Inc. v. United States*, 306 U.S. 226 (1939): "It was enough that, knowing that concerted action was contemplated or invited, the distributors gave their adherence to the scheme and participated in it. ... [A]cceptance by competitors, without previous agreement, of an invitation to participate in a plan, the necessary consequence of which, if carried out, is restraint of interstate commerce, is sufficient to establish an unlawful conspiracy under the Sherman Act."

[7]Rightfully and frequently, lawyers remind economists of our inadequacy in this regard:

> While properly applied economic science may allow an economist to reach conclusions about "collusion," the term as used by economists may include both tacit and overt collusion among competitors ... and it is unclear whether economists have any special expertise to distinguish between the kinds of "agreement." [Milne and Pace (2003), p. 36]
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> On the ultimate issue of whether behavior is the result of a contract, combination, or conspiracy, however, courts routinely prevent economists from offering an opinion, because economics has surprisingly little to say about this issue. [Page (2007), p. 424]

which both explicit and tacit collusion are feasible, when is collusion through explicit means significantly more profitable? To address those questions requires developing distinct theories of explicit collusion and tacit collusion. Of course, the primary challenge to modelling tacit collusion is dispensing with the assumption of equilibrium and allowing for less than full mutual understanding among firms.[8]

The contribution of this paper is in developing a theory of tacit collusion. Two essential elements of a model of tacit collusion are: 1) a transparent mechanism for coordinating on a collusive outcome; and 2) a plausible amount of mutual understanding among firms. The coordination mechanism considered here is price leadership, which is a commonly observed method of tacit collusion.[9] In terms of mutual understanding, it is assumed that it is common knowledge among firms that price increases will be at least matched and that failure to do so results in reversion to the pre-collusive outcome.[10] What is not common knowledge is leadership protocol. Which firm will lead by raising price? What price will it set? Is another firm expected to lead the next round of price hikes? In other words, there is mutual understanding among firms about the general mechanism of price leadership and price matching, but firms may lack common beliefs regarding the specific sequence of prices. Another way to view this assumption on mutual understanding is that, rather than suppose a strategy profile is common knowledge as is done with an equilibrium analysis, it is instead assumed to be common knowledge that firms' strategies lie in a subset of the strategy space. I will argue that this assumption on mutual understanding is plausibly achieved without express communication of the variety that would be a Section 1 violation.

Without the equilibrium assumption, two questions are of particular interest. First, can we characterize firms' prices when they lack mutual understanding as to their strategies? More broadly, how much mutual understanding is required to say something precise? Second, assuming we can say something precise, what is the

[8] One should not be misled to believe that the theoretical industrial organization literature is replete with theories of tacit collusion by virtue of use of the expression, as exemplified by the excellent survey "The Economics of Tacit Collusion" (Ivaldi et al, 2003). These theories characterize collusive behavior assuming full mutual understanding of strategies (that is, equilibrium) and are agnostic regarding how mutual understanding is reached. There is, however, some research that is most naturally considered explicit collusion because it assumes firms expressly communicate within the context of an equilibrium. Cheap talk messages about firms' private information on cost are exchanged in Athey and Bagwell (2001, 2008), on demand in Aoyagi (2002), Hanazono and Yang (2007), and Gerlach (2009), and on sales in Harrington and Skrzypacz (2011). There is also a body of work on bidding rings in auctions where participation in the auction is preceded by a mechanism among the ring members that involves the exchange of reported valuations; see, for example, Graham and Marshall (1987) and Krishna (2010).

[9] See Markham (1951) for an early discussion of price leadership and collusion, and Scherer (1980, Chapter 6) for several examples. In the equilibrium setting, some relevant papers exploring price leadership as a collusive device include Rotemberg and Saloner (1990) and Mouraviev and Rey (2011).

[10] The role of price-matching here is to coordinate on a collusive outcome. It has also been explored as a form of punishment; see Lu and Wright (2010) and Garrod (2011).

cost to firms from not having full mutual understanding? Is price lower under tacit collusion than if they were to engage in express communication and achieve the mutual understanding of strategies implicit in equilibrium?

In answer to the first question, I show that a precise statement can be made as to the steady-state price, though the transition path eludes characterization. As regards the second question, the lack of full mutual understanding does indeed constrain the extent of collusion; the steady-state price is strictly below the highest sustainable equilibrium price. In other words, if firms could expressly communicate, they would sustain a price in excess of that which is achieved under tacit collusion.

To my knowledge, the only other theory of tacit collusion is MacLeod (1985), whose approach is very different. To begin, it is based on firms announcing proposed price changes rather than making actual price changes. Axioms specify how firms respond to a price announcement, and these axioms are common knowledge. A firm's price response is allowed to depend on the existing price vector and the announced price change, and it is assumed the firm which announces the price change will implement it. If it is assumed that the price response is continuous with respect to the announcement, invariant to scale changes, and independent of firm identity then the response function must entail matching the announced price change.[11] When firms are symmetric, the theory predicts that the joint profit maximum is achieved. To the contrary, the theory developed here predicts price is always below the price that maximizes joint profit.

Of some relevance to the current paper is the literature on the rational learning of strategies in a repeated game; see, for example, Kalai and Lehrer (1993) and Nachbar (2005). The main result of Kalai and Lehrer (1993) is that if players are rational and each starts with a set of beliefs on other players' strategies that are compatible with the strategies actually chosen then play must converge in finite time to an $\varepsilon-$Nash equilibrium of the repeated game, for arbitrarily small ε. Assumptions are very weak in that a player need not know other players' payoffs or whether they are rational. In contrast, it is assumed here that rationality and payoff functions are common knowledge. While both that literature and the current paper explore behavior in a repeated game setting when strategies are not common knowledge, their objectives are very different. The rational learning literature seeks to determine how weak one can make the assumptions on beliefs in an infinitely repeated setting and still achieve convergence on an equilibrium. The current paper's goal is to develop a theory of tacit collusion; that is, making predictions on price based on plausible assumptions on mutual understanding. Given these distinct goals, the amount of structure placed on prior beliefs is very different. The rational learning literature only requires that a player's prior beliefs on the other players' strategies include their actual strategies

[11] If it was not assumed to be common knowledge that the firm announcing the price change would implement it then another price response function which satisfies the axioms is one which has a zero price response. In fact, it should be stated as a fourth axiom that the price change of the firm announcing the price change equals that announcement.

in the support. The paper here draws from the context of tacit collusion in a market to place a substantive, though plausible, amount of structure on prior beliefs, and then derive its implications for prices. The results are more precise but then the assumptions are stronger.[12]

The model is described in Section 2 - where standard assumptions are made regarding cost, demand, and firm objectives - and in Section 3 - where the assumption of equilibrium is replaced with alternative assumptions on the behavior and beliefs of firms. An upper bound on price under tacit collusion is derived in Section 4 which, by way of example, is shown in Section 5 cannot generally be improved upon. A modest additional assumption is made in Section 6 to precisely predict the long-run price produced by tacit collusion. Results are extended to when firms have different discount factors in Section 7. For the case of linear demand and cost functions, Section 8 explores the price effect of firms coordinating their behavior through tacit, rather than explicit, means. Section 9 offers a few concluding remarks.

2 Assumptions on the Market

Consider a symmetric differentiated products price game with n firms. $\pi(p_i, \mathbf{p}_{-i}) : \Re^n_+ \rightarrow \Re$ is a firm 's profit when it prices at p_i and its rivals price at $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$. Assume $\pi(p_i, \mathbf{p}_{-i})$ is bounded, twice continuously differentiable, increasing in a rival's price p_j $(j \neq i)$, and strictly concave in own price p_i. A firm's best reply function then exists:

$$\psi(\mathbf{p}_{-i}) = \arg\max_{p_i} \pi(p_i, \mathbf{p}_{-i}).$$

Further assume

$$\frac{\partial^2 \pi}{\partial p_i \partial p_j} > 0, \ \forall j \neq i$$

from which it follows that $\psi(\mathbf{p}_{-i})$ is increasing in p_j, $j \neq i$. A symmetric Nash equilibrium price, p^N, exists and is assumed to be unique,

$$\psi(p, .., p) \gtreqless p \text{ as } p \lesseqgtr p^N,$$

and let

$$\pi^N \equiv \pi\left(p^N, ..., p^N\right) > 0.$$

[12] In a related spirit, Wolitsky (2011) derives a lower bound on a player's payoff in an infinite horizon bargaining setting when player i knows: 1) player j is rational; and 2) player j knows that player i is committed to a particular strategy (referred to as a "posture") with probability ε. When the player's posture is chosen strategically, the lower bound on a player's payoff is characterized and is shown to be large relative to ε.

Assuming $\pi(p,...,p)$ is strictly concave in p, there exists a unique joint profit maximum p^M,

$$\sum_{j=1}^{n} \frac{\partial \pi(p,...,p)}{\partial p_j} \gtreqqless 0 \text{ as } p \lesseqqgtr p^M,$$

and $p^M > p^N$.

Firms interact in an infinitely repeated price game with perfect monitoring. A collusive price $p' > p^N$ is sustainable with the grim trigger strategy if and only if:[13]

$$\left(\frac{1}{1-\delta}\right)\pi(p',...,p') \geq \max_{p_i<p} \pi(p_i,p',...,p') + \left(\frac{\delta}{1-\delta}\right)\pi^N, \tag{1}$$

where δ is the common discount factor.[14] Define $\widetilde{p}$ as the best price sustainable using the grim trigger strategy:

$$\widetilde{p} \equiv \max\left\{p \in \left[p^N, p^M\right] : \left(\frac{1}{1-\delta}\right)\pi(p,...,p) \geq \max_{p_i<p} \pi(p_i,p,...,p) + \left(\frac{\delta}{1-\delta}\right)\pi^N\right\}.$$

Assume $\widetilde{p} > p^N$ and if $\widetilde{p} \in \left(p^N, p^M\right)$ then

$$\left(\frac{1}{1-\delta}\right)\pi(p,...,p) \gtreqqless \pi(\psi(p,...,p),p,...,p) + \left(\frac{\delta}{1-\delta}\right)\pi^N \text{ as } p \lesseqqgtr \widetilde{p} \text{ for } p \in \left[p^N, p^M\right]. \tag{2}$$

$\widetilde{p}$ will prove to be a useful benchmark.

For the later analysis, consider the "price matching" objective function for a firm:

$$W(p_i, \mathbf{p}_{-i}) \equiv \pi(p_i, \mathbf{p}_{-i}) + \left(\frac{\delta}{1-\delta}\right)\pi(p_i,...,p_i).$$

Given its rivals price at $\mathbf{p}_{-i}$ in the current period, $W(p_i, \mathbf{p}_{-i})$ is firm i's payoff from pricing at p_i *if it believed* that all firms would match that price in all ensuing periods. Consider

$$\frac{\partial W(p_i, \mathbf{p}_{-i})}{\partial p_i} = \frac{\partial \pi(p_i, \mathbf{p}_{-i})}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi(p_i,...,p_i)}{\partial p_j}.$$

If $p_i < p^M$ then the second term is positive; by raising its current price, a firm increases the future profit stream under the assumption that its price increase will be matched by its rivals. If $p_i > \psi(p_{-i})$ then the first term is negative. Evaluate $\frac{\partial W(p_i,p_{-i})}{\partial p_i}$ when firms price at a common level p:

$$\begin{aligned} \frac{\partial W(p,...,p)}{\partial p_i} &= \frac{\partial \pi(p,...,p)}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi(p,...,p)}{\partial p_j} \\ &= \left(\frac{1}{1-\delta}\right)\left(\frac{\partial \pi(p,...,p)}{\partial p_i} + \delta\sum_{j\neq i}^{n}\frac{\partial \pi(p,...,p)}{\partial p_j}\right) \end{aligned}$$

[13] The grim trigger strategy has any deviation from the collusive price p' result in a price of p^N forever.

[14] Section 7 shows that results are robust to when firms have different discount factors.

Thus, when $p \in \left(p^N, p^M\right)$, raising price lowering current profit, $\frac{\partial \pi(p,...,p)}{\partial p_i} < 0$, and increases future profit, $\sum_{j=1}^{n} \frac{\partial \pi(p,...,p)}{\partial p_j} > 0$. By the preceding assumptions, $W\left(p_i, \mathbf{p}_{-i}\right)$ is strictly concave in p_i since it is the weighted sum of two strictly concave functions. Hence, a unique optimal price exists,

$$\phi\left(\mathbf{p}_{-i}\right) = \max_{p_i} W\left(p_i, \mathbf{p}_{-i}\right). \tag{3}$$

$\phi\left(\mathbf{p}_{-i}\right)$ is referred to as the price matching best reply function for a firm. By the preceding assumptions, $\phi\left(\mathbf{p}_{-i}\right)$ is increasing in a rival's price as

$$\frac{\partial \phi\left(\mathbf{p}_{-i}\right)}{\partial p_j} = -\frac{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i \partial p_j}{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i^2} = -\frac{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i^2} + \left(\frac{\delta}{1-\delta}\right)\left(\frac{\partial^2 \pi(p,...,p)}{\partial p^2}\right)} > 0.$$

As there is a benefit in terms of future profit from raising price (as long as it does not exceed the joint profit maximum) then the price matching best reply function results in a higher price than the standard best reply function. To show this result, consider

$$\begin{aligned}\frac{\partial W\left(\psi\left(\mathbf{p}_{-i}\right), \mathbf{p}_{-i}\right)}{\partial p_i} &= \frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right), \mathbf{p}_{-i}\right)}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n} \frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right), ..., \psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j} \\ &= \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n} \frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right), ..., \psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j} > 0,\end{aligned}$$

which is positive because $\mathbf{p}_{-i} \leq \left(p^M, ..., p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right) < p^M$.[15] By the strict concavity of W, $\phi\left(\mathbf{p}_{-i}\right) > \psi\left(\mathbf{p}_{-i}\right)$.

ϕ has a fixed point p^* because it is continuous, $\phi\left(p^N, ..., p^N\right) > p^N$, and

$$\frac{\partial W\left(p^M, ..., p^M\right)}{\partial p_i} = \frac{\partial \pi\left(p^M, ..., p^M\right)}{\partial p_i} < 0 \Rightarrow \phi\left(p^M, ..., p^M\right) < p^M.$$

Further assume the fixed point is unique:

$$\phi\left(p, ..., p\right) \gtreqless p \text{ as } p \lesseqgtr p^*.$$

Thus, if rival firms price at p^*, a firm prefers to price at p^* rather than price differently under the assumption that its price will be matched forever. p^* will prove to be a useful benchmark.

Results are proven when the price set is finite.[16] From hereon, assume the price set is $\Delta_\varepsilon \equiv \{0, \varepsilon, 2\varepsilon, ..., \}$, where $\varepsilon > 0$ and is presumed to be small. For convenience,

[15] Since $\psi\left(p, .., p\right) \gtreqless p$ as $p \lesseqgtr p^N$ then $\psi\left(p^M, .., p^M\right) < p^M$. Given that ψ is increasing then $\mathbf{p}_{-i} \leq \left(p^M, ..., p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right) < \psi\left(p^M, .., p^M\right) < p^M$.

[16] A discussion of the case of an infinite price set is provided at the end of Section 4.

suppose $p^N, p^*, \widetilde{p} \in \Delta_\varepsilon$.[17] As the finiteness of the price set could generate multiple optima, define the best reply correspondence for the price matching objective function:

$$\overline{\phi}(\mathbf{p}_{-i}) \equiv \arg\max_{p_i \in \Delta_\varepsilon} \pi(p_i, \mathbf{p}_{-i}) + \left(\frac{\delta}{1-\delta}\right) \pi(p_i, ..., p_i).$$

The best reply correspondence is assumed to have the following property:[18]

$$\overline{\phi}(\mathbf{p}_{-i}) \begin{cases} \subseteq \{p' + \varepsilon, ..., p^*\} & \text{if } \mathbf{p}_{-i} = (p', ..., p') \text{ where } p' < p^* - \varepsilon \\ = \{p^*\} & \text{if } \mathbf{p}_{-i} = (p_i^*, ..., p_i^*) \\ \subseteq \{p^*, ..., p' - \varepsilon\} & \text{if } \mathbf{p}_{-i} = (p', ..., p') \text{ where } p' > p^* + \varepsilon \end{cases} \tag{4}$$

Recall that p^* is the unique fixed point for $\phi(\mathbf{p}_{-i})$ and is also a fixed point for $\overline{\phi}(\mathbf{p}_{-i})$. By (4), if all rival firms price at p' then firm i's best reply has it price above p' when $p' < p^* - \varepsilon$. Analogously, if $p' > p^* + \varepsilon$ then firm i's best reply has it price below p'. Note that an implication of (4) is that the set of symmetric fixed points of $\overline{\phi}(\mathbf{p}_{-i})$ is, at most, $\{p^* - \varepsilon, p^*, p^* + \varepsilon\}$.

The example case of linear demand and cost functions in Section 8 satisfies all of the assumptions made in Section 2.

3 Assumptions on Beliefs and Behavior

The equilibrium approach to characterizing firm pricing entails making assumptions on behavior - each firm acts to maximize its payoff given the conjectured strategies of the other firms - and beliefs - each firm's conjectures are accurate. The standard behavioral assumption is retained by assuming firms are rational, firms believe other firms are rational, and so forth.

Assumption A1: A firm is rational in the sense of choosing a strategy to maximize the present value of its expected profit stream given its beliefs on other firms' strategies, and rationality is common knowledge.

As the focus here is on tacit collusion - in which case firms do not engage in express communication - assuming firms have accurate beliefs as to their rivals' strategies is problematic, especially in light of the abundance of collusive equilibria. The approach taken here is to weaken the equilibrium assumption that the strategy profile is common knowledge by instead assuming that only some properties of firms' strategies are common knowledge. Alternatively stated, it is common knowledge that firms' strategies lie in a subset of the strategy space; equilibrium is when the subset is a singleton.

[17] If $\widetilde{p} \in \Delta_\varepsilon$ and $\psi(\widetilde{p}, ..., \widetilde{p}) \in \Delta_\varepsilon$ then $\widetilde{p}$ is still the best price sustainable using the grim punishment.

[18] It is shown in Appendix A that a sufficient condition for (4) is $-\frac{\partial^2 \pi}{\partial p_i^2} \geq 2\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}$, which holds when demand and cost functions are linear.

Definition: The strategy of firm i has the *price matching plus* (PMP) property if:

$$p_i^t \begin{cases} \in \left\{ \max\left\{p_1^{t-1},...,p_n^{t-1}\right\},...,\widetilde{p}\right\} & \text{if } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\widetilde{p}\right\} \ \forall j, \forall \tau \leq t-1 \\ & \text{and } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} < \widetilde{p} \\ & \\ = \widetilde{p} & \text{if } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\widetilde{p}\right\} \ \forall j, \forall \tau \leq t-1 \\ & \text{and } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} \geq \widetilde{p} \\ & \\ = p^N & \text{if not } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\widetilde{p}\right\} \\ & \forall j, \forall \tau \leq t-1 \end{cases}$$

First note that matching a price increase means setting $p_i^t = \max\left\{p_1^{t-1},...,p_n^{t-1}\right\}$. Thus, as of period t, price increases have always been *at least* matched when $p_j^\tau \geq \max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\}$ $\forall j, \forall \tau \leq t-1$. The PMP property has firms only price as high as $\widetilde{p}$, where recall that $\widetilde{p}$ is the highest equilibrium price using the grim punishment. Thus, firms have been complying with this modified price matching when $p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\widetilde{p}\right\}$ $\forall j, \forall \tau \leq t-1$. In that event, the PMP property has a firm price in period t at least as high as $\max\left\{p_1^{t-1},...,p_n^{t-1}\right\}$ with the caveat of not pricing in excess of $\widetilde{p}$. Finally, if any firm should fail to act in a manner consistent with this price matching behavior then a firm will revert to pricing at the non-collusive price p^N thereafter. A strategy satisfying the PMP property will be referred to as being PMP-compatible.

Assumption A2: It is common knowledge that a firm's strategy satisfies the price matching plus property.

By A2, there is a "meeting of the minds" among firms that: 1) price increases are at least matched as long as past price increases have been at least matched in the past; 2) price increases will be followed only as high as $\widetilde{p}$; and 3) departure from this price matching behavior results in reversion to non-collusive pricing. I now want to argue how mutual understanding among firms that their strategies have these properties could plausibly be achieved without express communication of the sort associated with explicit collusion. Each of the three properties will be taken in turn.

Let us start by examining how it could become common knowledge that price increases will at least be matched. First, it could occur through unilateral public announcements whereby one firm's manager declares elements of a strategy that encompasses price leadership and price matching. For example, in the one-way truck rental market, the FTC claimed that, during a public announcement regarding earnings, the CEO of U-Haul repeatedly emphasized that U-Haul was demonstrating "price leadership" and was "trying to force prices."[19] Second, mutual understanding could be achieved by the adoption of actions that served to communicate an expectation among firms that they will engage in coordinated pricing. It is argued in

[19] *Matter of U-Haul Int'l Inc. and AMERCO* (FTC File No. 081-0157, July 10, 2010).

Harrington (2011) that, under certain market conditions, the mutual adoption of the posted price format signals that firms expect to collude. In the case of the turbine generator market, General Electric and Westinghouse mutually adopted the posted price format and subsequently engaged in tacit collusion through price leadership and price matching; there was no evidence of any express communication.[20] Thus, by taking certain costly actions that would only be optimal if firms did engage in tacit collusion, a common expectation of price matching was achieved. Third, mutual understanding of price matching could be acquired by way of example. One firm could raise its price and if rivals subsequently matched that price then firms may then have mutual understanding regarding price matching; from that point onward, Assumption A2 could well hold. This is a view that has been expressed by Richard Posner, first as a scholar and then as a judge in the High Fructose Corn Syrup case:

> [O]ne seller communicates his "offer" by restricting output, and the offer is "accepted" by the actions of this rivals in restricting their outputs as well. It may therefore be appropriate in some cases to instruct a jury to find an agreement to fix prices if it is satisfied that there was a tacit meeting of the minds of the defendants on maintaining a noncompetitive pricing policy.[21]
>
> If a firm raises price in the expectation that its competitors will do likewise, and they do, the firm's behavior can be conceptualized as the offer of a unilateral contract that the offerees accept by raising their prices.[22]

In summary, there are a variety of indirect forms of communication that could result in firms having mutual understanding regarding price matching.

Next, consider the assumption that it is common knowledge that failure to at least match price increases (up to a maximum price of $\widetilde{p}$) results in non-collusive pricing thereafter. If, in fact, a firm acts to the contrary - by not following a price increase or undercutting price - then observed behavior will run contrary to common expectations. In describing how firms respond to this incongruity between beliefs and behavior, I will draw on Lewis (1969) to argue that firms resort to an outcome that is perceived as *salient*. Lewis (1969) defines a salient outcome as "one that stands out from the rest by its uniqueness in some conspicuous respect"[23] and that precedence is one source of saliency: "We may tend to repeat the action that succeeded before if we have no strong reason to do otherwise."[24] Cubitt and Sugden (2003) stress the latter qualifier and note that "precedent allows the individual to make inductive inferences

[20] The posted price format has firms publicly announce a non-negotiable price. For details on the turbine generator case, see Scherer (1980, p. 182) and Hay (2000).

[21] Posner (2001), pp. 94-95.

[22] *In Re High Fructose Corn Syrup Antitrust Litigation Appeal of A & W Bottling Inc et al*, U.S. Court of Appeals, 295 F3d 652, (7th Cir., 2002).

[23] Lewis, (1969), p. 35.

[24] Lewis, (1969), p. 37.

in which she has *some* confidence, but which are overridden whenever deductive analysis points clearly in a different direction."[25]

With this perspective in mind, the movement from competition to tacit collusion can be seen as a shift from inductive to deductive reasoning. Firms have been competing and, by induction, they would expect to continue to do so. However, either through price signaling or public announcement of strategies or some other coordinating event, firms supplant inductive inferences with deductive reasoning so that a common expectation of competition is replaced with a common expectation of price matching. With this as a backdrop, my claim is that a subsequent departure in behavior from price matching implies a breakdown in the efficacy of deductive reasoning, in response to which firms revert to the original inductive analysis and therefore the competitive solution. Here I am appealing to the view that firms will "tend to pick the salient as a last resort."[26] The saliency of the competitive solution emanates from it being the most recent outcome (prior to the current episode of tacit collusion) that was common knowledge to firms.[27]

There are two implicit assumptions in the preceding argument that warrant discussion. First, the saliency of the competitive solution relies on it prevailing prior to this episode of tacit collusion. However, that is not essential for the paper's main results. If some other behavior described the pre-collusion setting then that behavior can be assumed instead. What *is* critical is that how firms respond to the departure from price matching is common knowledge and the associated continuation payoff is lower than if firms had abided by the PMP property.[28] A second assumption, which figures prominently in discussions of saliency (such as in Lewis, 1969), is that the current post-collusion situation is sufficiently similar to the pre-collusion situation so that induction on the latter is compelling. It is well-recognized that

> no two interactions are exactly alike. Any two real-world interactions will differ in matters of detail, quite apart from the inescapable fact that "previous" and "current" interactions occur at different points in time. Thus, the idea of "repeating what was done in previous instances of the game" is not well-defined. Precedent has to depend on analogy: to follow precedent in the presence instance is to behave in a way that is *analogous with* behaviour in past instances. ... Inductive inference is possible only because a very small subset of the set of possible patterns is privileged.[29]

[25] Cubitt and Sugden (2003), p. 196. Also see Sugden (2011).

[26] Lewis, (1969), p. 35.

[27] If two people are physically separated and in search of each other, a salient place for them to meet is the last place that they were together. That place is common knowledge to them - as they both witnessed each other there - and it is singular in being the most recent place visited that is common knowledge. Analogously, if there is inconsistency in firm behavior, firms may return to the most recent strategy profile that was common knowledge.

[28] While the reversion to some non-collusive outcome is motivated by its saliency, it will serve the usual role as a punishment in response to non-compliant behavior.

[29] Cubitt and Sugden (2003), pp. 196-7.

The post-collusion scenario most notably differs from the pre-collusion scenario in that the former was preceded by an episode of collusion, while the latter was (probably) not. Though this difference could disrupt the saliency of the pre-collusion outcome when it comes to responding to a departure from the PMP property, it is reasonable for its saliency to remain intact which is the presumption made here.

The third and final feature to the PMP property is that a firm will not price in excess of $\widetilde{p}$, which means that it will follow price increases only as high as $\widetilde{p}$ and, as a price leader, will not raise price beyond $\widetilde{p}$. It is surely compelling for a firm to have some upper bound to how high it will price. For the purpose of this discussion, denote this upper bound to be $\overline{p}$ and let $\overline{p}$ replace $\widetilde{p}$ in the PMP property. Next, define p^U as the highest price satisfying (1); note that it can exceed p^M. Now suppose $\overline{p} > p^U$. If $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} = \overline{p}$ then, by A2, $p_j^\tau = \overline{p}\ \forall j, \forall \tau \geq t$. However, this pricing behavior contradicts A1 as it follows from $\overline{p} > p^U$ that

$$\left(\frac{1}{1-\delta}\right)\pi\left(\overline{p}, ..., \overline{p}\right) < \pi\left(\psi\left(\overline{p}, ..., \overline{p}\right), \overline{p}, ..., \overline{p}\right) + \left(\frac{\delta}{1-\delta}\right)\pi^N,$$

which implies a firm does better by pricing at $\psi\left(\overline{p}, ..., \overline{p}\right)$ and earning π^N thereafter (which it can expect by A2). Hence, A1-A2 imply that $\overline{p} \leq p^U$. It would also seem reasonable to suppose that $\overline{p} \leq p^M$ so firms do not follow price increases beyond the joint profit maximizing price. In that case, $\overline{p} \leq \widetilde{p}$.

That it is common knowledge that price increases will not be matched beyond what is consistent with rationality ($\overline{p} \leq p^U$) is compelling, and beyond what is commonly recognized as most desirable ($\overline{p} \leq \widetilde{p}$) is reasonable. But A2 goes further in specifying that $\overline{p} = \widetilde{p}$. One argument for this property is from a collective rationality perspective in that following price increases all the way up to $\widetilde{p}$ is best for all firms. In fact, this property would seem quite convincing for when there are two firms. If firm 1 raised price in the previous period to $\widetilde{p}$ then it is in the interests of firm 2 to match that price as long as it expects firm 1 to do so. Since firm 1 was the one which raised price to $\widetilde{p}$, it has revealed a willingness to price at $\widetilde{p}$ which makes such a belief for firm 2 quite reasonable. However, such an argument does not extend to when there are three or more firms. Even if firm 1 raised price in the previous period to $\widetilde{p}$, firm 2 may not match that price because it is uncertain whether firm 3 will match it, and firm 3 may not match it because it is uncertain firm 2 will match it. While firm 1 has revealed a preference for pricing at $\widetilde{p}$, firms 2 and 3 have not. Recognizing this concern, it still seems plausible that it could be common knowledge among firms that price increases will be matched up to $\widetilde{p}$.

Let me summarize the assumptions on behavior and beliefs. In terms of behavior, it is assumed that a firm is rational, a firm will (at least) match a rival's price as long as price does not exceed the highest sustainable price, and a firm will revert to competitive pricing if any firm should depart from this price matching behavior. The restriction on beliefs is that this behavior is common knowledge. Consistent with tacit collusion, it has been argued that this is a plausible amount of mutual understanding that could reasonably be achieved without the express communication associated with explicit collusion. Furthermore, there remains significant residual uncertainty among firms about the strategies of their rivals. It is not common knowledge as to who will lead a price increase, when it will occur, what price a leader will set, and whether price increases will just be matched or instead exceeded.

In Section 4, it is shown that these assumptions are sufficient to place a non-trivial upper bound on price. Section 5 provides an example to show that more structure is required to tighten the bound on pricing. In Section 6, some minimal additional structure on beliefs is added to precisely characterize the steady-state price under tacit collusion.

4 An Upper Bound on Price under Tacit Collusion

To begin the analysis, it is essential to show that Assumptions A1 and A2 are compatible in the sense that a firm's best reply is a PMP-compatible strategy when it believes its rivals use PMP-compatible strategies. After showing that a rational firm uses a PMP-compatible strategy, Theorem 2 presents the main result of this section which is to offer an upper bound on price under tacit collusion. Theorem 3 shows that this upper bound is strictly below the best equilibrium price.

Define S^{PMP} as the subset of a firm's strategy space that satisfies the PMP property. Theorem 2 shows that if firm i's beliefs over other firms' strategies have support in S^{PMP} then firm i's best reply must lie in S^{PMP}; that is, the set S^{PMP} is closed under the best reply operator. All proofs are in Appendix B.

Lemma 1 *Assume* $\max\{p_1^0, ..., p_n^0\} \geq p^N$*. If firm i's beliefs over other firms' strategies have support in S^{PMP} then, for all histories, firm i's best reply lies in S^{PMP}.*

Theorem 2 shows that if it is common knowledge that firms are rational and that firms use strategies satisfying the PMP property then price is bounded above by (approximately) p^*.[30]

Theorem 2 *Assume A1-A2. If* $(p_1^0, ..., p_n^0) = \left(p^N, ..., p^N\right)$ *then* $\{(p_1^t, ..., p_n^t)\}_{t=1}^{\infty}$ *is weakly increasing over time and there exists finite T such that* $p_1^t = \cdots = p_n^t = \widehat{p}$ $\forall t \geq T$ *where* $\widehat{p} \leq p^* + \varepsilon$*.*

In explaining the basis for Theorem 2, first note that while A2 leaves unspecified whether some firm will initiate a price increase, it is fully consistent with A1-A2 for a firm to be a price leader. For example, if a firm believed other firms would not

[30]Theorem 2 is stated for when firms are initially pricing competitively, which is the appropriate starting point if firms are moving from competition to tacit collusion. The result, however, trivially extends to when $\left(p_1^0, ..., p_n^0\right) \in \left\{p^N, ..., p^* + \varepsilon\right\}^n$.

raise price then it would be rational for this firm to increase price (as long as the current price is not too high). The issue is how far would it go in raising price. If the firm expected that its price increase would only be met and never exceeded by a rival (for example, rivals are believed to only match price increases) then it would not want to raise price above (approximately) p^*.[31] Recall that p^* is the price at which a firm, if it were to raise price from p^* to any higher level (call it p') then it would lose more in current profit (because of lower demand from pricing above the level p^* set by its rivals) than it would gain in future profit (from all firms pricing at p'). Thus, a firm that believed its rivals would never initiate price increases would not raise price beyond p^*. However, a firm might be willing to lead a price increase above p^* *if* it believed it would induce a rival to further increase price; for example, if the firm believed that firms would take turns leading price increases. The essence of the proof of Theorem 2 is showing that cannot happen.

To begin, a firm will never price above $\widetilde{p}$, which is the minimum of the highest sustainable price and the joint profit maximum. If $\widetilde{p} > p^*$ then it furthermore means that a firm would never raise price *to* $\widetilde{p}$ since such a price increase would only induce its rivals to match that price; it would not induce them to further raise price. Thus, if a firm is rational and it believes the other firms use PMP-compatible strategies then it will not raise price to $\widetilde{p}$. This puts an upper bound on price of $\widetilde{p} - \varepsilon$. We next build on that result to argue that $\widetilde{p} - 2\varepsilon$ is an upper bound on price. Given it is common knowledge that firms are rational and firms use PMP-compatible strategies, firm i then believes firm $j\ (\neq i)$ is rational and also that firm j believes firm h uses a PMP-compatible strategy (for all $h \neq j$); hence, firm i knows that firm j will not raise price to $\widetilde{p}$. This means that firm i knows that if it raises price to $\widetilde{p} - \varepsilon$ that this price increase will only be matched and not exceeded, which then makes a price increase to $\widetilde{p} - \varepsilon$ unprofitable (as long as $\widetilde{p} - \varepsilon > p^*$). Given that all firms are not willing to raise price to $\widetilde{p} - \varepsilon$ then $\widetilde{p} - 2\varepsilon$ is an upper bound on price. The proof is completed by induction - with each step using another layer of common knowledge in A1 and A2 - to end up with the conclusion that a firm would never raise price to a level exceeding $p^* + \varepsilon$. Hence, price is bounded above by (approximately) p^*.

In deriving this upper bound, the punishment for deviation from (at least) matching price is reversion to a stage game Nash equilibrium. Such a punishment could, in principle, sustain a price as high as $\widetilde{p}$. The next result shows that tacit collusion fall short because the upper bound on price under tacit collusion is strictly less than the highest sustainable equilibrium price.

Theorem 3 $p^* \in \left(p^N, \widetilde{p}\right)$.

Recall that p^* is the price at which the reduction in current profit from a marginal increase in a firm's price is exactly equal in magnitude to the rise in the present value

[31] In discussing results, I will generally refer to the upper bound as p^* rather than $p^* + \varepsilon$ since ε is presumed to be small.

of the future profit stream when that higher price is matched by all firms for the infinite future. Equivalently, p^* is the price at which the increase in current profit from a marginal *decrease* in price to $p^* - \varepsilon$ is exactly equal in magnitude to the *fall* in the present value of the future profit stream *when the firm's rivals lower price to* $p^* - \varepsilon$ (when ε is small). In comparison, $\widetilde{p}$ is the price for a firm at which the increase in current profit from a marginal decrease in price is exactly equal in magnitude to the fall in the present value of the future profit stream *when the firm's rivals lower price to* p^N.[32] Given that the punishment is more severe in the latter case, it follows that the maximal sustainable price is higher: $\widetilde{p} > p^*$.

Under tacit collusion, the steady-state price is bounded above by p^* even though higher prices are sustainable. In other words, if firms started at a price of $\widetilde{p}$ then such a price would persist. But if firms start with prices below p^*, such as at the non-collusive price p^N, then prices will not go beyond p^*, even though higher prices are sustainable. The obstacle is that it is not in the interests of any firm to lead a price increase beyond p^*. Note that Theorems 2 and 3 are robust to the form of the punishment. p^* is independent of the punishment and, given another punishment, $\widetilde{p}$ would just be the highest sustainable price with that punishment. In particular, if the punishment is at least as severe as the grim punishment then Theorems 3 and 4 are unchanged.

In concluding, let me discuss the role of the finiteness of the price set. p^* is the highest price to which a firm will raise price if it can only anticipate that other firms will match its price. Thus, a firm is willing to take the lead and price above p^* only if, by doing so, it induces a rival to enact further price increases. Since no firm will price above $\widetilde{p}$ then raising price to $\widetilde{p}$ cannot induce rivals to lead future price increases. Thus, a firm will not raise price to a level beyond $\widetilde{p}-\varepsilon$, which means $\widetilde{p}-\varepsilon$ is an upper bound on price. This argument works iteratively to ultimately conclude that p^* is (approximately) an upper bound on price. The finiteness of price is critical in this proof strategy for it allows $\widetilde{p} - \varepsilon$ to be well-defined. However, even with an infinite price set, it is still the case that a necessary condition for a firm to lead and raise price above p^* is that it will induce a rival to enact further price increases. As that must always be true then, if the limit price exceeds p^* when there is an infinite price set, price cannot converge in finite time. But since it is still the case that $\widetilde{p}$ is an upper bound on price, the price increases must then get arbitrarily small; eventually, each successive price increase will bring forth a smaller future price increase by a rival. I am not arguing that this argument will prevent Theorem 2 from extending to the infinite price set but rather that it is the only argument that could possibly do so. In sum, either Theorem 2 extends to when the price set is infinite or, if it does not, then it implies a not very credible price path with never-ending price increases that eventually become arbitrarily small. The oddity of such a price path would seem an artifact of assuming an infinite set of prices when, in fact, the set of prices is finite.

[32] That is, $\widetilde{p}$ is the highest price for which a firm incentive compatibility constraint, (1), holds. For this discussion, suppose $\widetilde{p} < p^M$.

5 Example: Price Can be Competitive or Supra-competitive

By Theorem 2, if it is common knowledge that firms are rational and that their strategies satisfy the PMP property then price is bounded above by p^*. But is p^* the least upper bound? And is there a lower bound on price exceeding p^N? The purpose of the current section is to show, by way of example, that it is consistent with A1-A2 for price to converge to p^* and also fail to rise above p^N.Thus, a tighter result than Theorem 2 will require additional assumptions, which is the objective of the next section.

For the duopoly case, suppose the price set is composed of just three elements, $\{p^N, p', p^M\}$, and $p' \equiv \left(p^M + p^N\right)/2$. Assume δ is sufficiently close to one which has the implication: $p^* = \widetilde{p} = p^M$.[33] Consider the following pair of functions which map from the lagged maximum price to current price:

$$\begin{aligned} S^L\left(p^N\right) &= p', S^L\left(p'\right) = p^M, S^L\left(p^M\right) = p^M \\ S^F\left(p^N\right) &= p^N, S^F\left(p'\right) = p', S^F\left(p^M\right) = p^M \end{aligned} \tag{5}$$

S^L (where L denotes "leader") has a firm raise price to p' when the lagged maximum price is p^N, to p^M when the lagged maximum price is p', and price at p^M when the lagged maximum price is p^M. S^F (where F denotes "follower") has a firm's price equal the lagged maximum price. When S^L (or S^F) is referred to as a strategy, it is meant that the specification in (5) applies when both firms have priced at least as high as the previous period's maximum price in all past periods, and otherwise a firm prices at p^N. Thus, these strategies satisfy the PMP property. It is shown in Appendix C that $\left(S^L, S^F\right)$ is a subgame perfect equilibrium when $\delta \simeq 1$ and

$$\pi\left(p', p^N\right) + \pi\left(p^M, p'\right) > \pi\left(p^M, p^N\right) + \pi\left(p^M, p^M\right). \tag{6}$$

It is also shown that (6) holds for the case of linear demand and cost when products are sufficiently differentiated and/or cost is sufficiently low.

If $\left(S^L, S^F\right)$ is a subgame perfect equilibrium then it immediately follows that both S^L and S^F are rationalizable strategies (that is, consistent with A1). Thus, a price path of $\left(\left(p', p^N\right), \left(p^M, p'\right) \left(p^M, p^M\right), ...\right)$, with a steady-state price of p^M, is consistent with A1-A2. It is achieved by having firm 1 use S^L based upon the belief that firm 2 use S^F, and firm 2 uses S^F based upon the belief that firm 1 uses S^L; and these beliefs are consistent with A1. However, it is also the case that a price path of $\left(\left(p^N, p^N\right), ...\right)$ is consistent with A1-A2. It occurs when each firm uses S^F based upon the belief that the other firm uses S^L, and these beliefs are also consistent with A1. Thus, A1-A2 could produce supracompetitive prices or competitive prices.

[33] In this example, as opposed to elsewhere in the paper, p^* is defined for when the feasible price set is $\{p^N, p', p^M\}$ rather than $\Re_+$. This, however, is a good approximation when $\delta \simeq 1$ as then $p^* \simeq p^M$ when the price set is $\Re_+$. Of course, if $\delta \simeq 1$ then $\widetilde{p} = p^M$ whether the price set is $\{p^N, p', p^M\}$ or $\Re_+$.

6 Steady-State Price under Tacit Collusion

Thus far, assumptions have been made on a firm's beliefs regarding price matching - specifically, other firms will at least match price up to a maximum level of $\widetilde{p}$ - and regarding what happens when behavior is contrary to such price matching - other firms will revert to competitive prices. Of some importance is that no assumptions have been made regarding price leadership. In some markets, a particular firm may be the salient leader by virtue of its size or access to information (what is referred to as barometric price leadership; see, for example, Cooper, 1997). However, keep in mind that leadership is costly in that a firm that initiates a price hike will lose demand prior to its price being matched.[34] Hence, each firm would prefer another firm to take the lead in raising price and this could well result in a lack of common knowledge as to who will lead - as exemplified in the preceding section - as well as the price to be set. In light of this discussion, it would seem problematic to assume the identity of the price leader and the pattern of price increases to be common knowledge, at least in the absence of express communication. As shown below, a minimal and straightforward assumption about price leadership will prove sufficient to show that price converges to the upper bound of p^* identified in Theorem 2.

Define a *price path* to be an infinite sequence of price vectors, where each vector has n prices, one for each firm; a price path is then an outcome to the game. Define Ω as the set of price paths consistent with A1-A2 and $\Omega(p)$ as the subset of Ω that converge to p (necessarily in finite time). $\Omega(p,t)$ is composed of the price paths in $\Omega(p)$ through period $t-1$. Define $S_{-i}(p,h^t)$ to be the set of strategy profiles for all firms but i that are consistent with A1-A2 and history h^t, and that have firms price at p when the maximum lagged price is p (thus, they are a subset of the strategy profiles that converge to p).

Assumption A3: $\exists$ finite T such that if $h^t \in \Omega(p,t)$ and $p_j^\tau = p\ \forall j, \forall \tau = t-1-T, ..., t-1$ then firm i believes the other firms' strategy profile lies in $S_{-i}(p,h^t)$.

By A3, if the history is consistent with other firms using strategies that converge to p and if all firms have priced at p for a sufficiently long time then a firm believes that its rivals are using strategies that converge to p. Note that when $\Omega(p) = \varnothing$, so there

[34] Wang (2009) provides indirect evidence of the costliness of price leadership. In a retail gasoline market in Perth, Australia, Shell was the price leader over 85% of the time until a new law increased the cost of price leadership, after which the three large firms - BP, Caltex, and Shell - much more evenly shared the role of price leader. The law specified that every gasoline station was to notify the government by 2pm of its next day's retail prices, and to post prices on its price board at the start of the next day *for a duration of at least 24 hours*. Hence, a firm which led in price could not expect its rivals to match its price until the subsequent day. The difference between price being matched in an hour and in a day is actually quite significant given the high elasticity of firm demand in the retail gasoline market. For the Quebec City gasoline market, Clark and Houde (2011, p. 20) find that "a station that posts a price more than 2 cents above the minimum price in the city loses between 35% and 50% of its daily volume."

are no strategies consistent with A1-A2 that converge to p, A3 is vacuous and thus imposes no restrictions.[35]

Theorem 4 *If A1-A3 and* $(p_1^0, ..., p_n^0) = \left(p^N, ..., p^N\right)$ *then* $\{(p_1^t, ..., p_n^t)\}_{t=1}^{\infty}$ *is weakly increasing over time and there exists finite* T *such that* $p_1^t = \cdots = p_n^t = \widehat{p}\ \forall t \geq T$ *where* $\widehat{p} \in \{p^* - \varepsilon, p^*, p^* + \varepsilon\}$ *and* p^* *is defined by*

$$\frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_i} + \delta \sum_{j \neq i}^{n} \frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_j} = 0.$$

By A3, if prices have remained at the same level for a sufficiently long time then a firm believes the other firms are using strategies that converge to that price. As then it doesn't expect other firms to lead another price increase, if a further price increase is in fact profitable then a rational firm will enact it. As shown by the example in the preceding section, this assumption (or something similar) is necessary to ensure that firms do not end up in an infinitely long coordination failure whereby each firm doesn't raise price because it expects another firm to do so, in spite of an ever-growing history to the contrary.

The main contribution of Theorem 4 is offering a precise characterization of the steady-state price under tacit collusion while making modest assumptions on firms' behavior and beliefs. Though there is mutual understanding regarding rationality and price matching, nothing is common knowledge concerning price leadership, and a minimal condition is placed on a firm's beliefs as to who will lead. It is then possible to place restrictions on beliefs that are plausibly consistent with the absence of express communication and still describe where tacit collusion will take price in the long-run.

7 Generalization to Heterogeneous Discount Factors

The analysis has considered when firms are identical but results can be easily extended to when they have different discount factors. Letting δ_i denote the discount factor of firm i, assume

$$0 < \delta_n \leq \delta_{n-1} \leq \cdots \leq \delta_1 < 1.$$

The best price sustainable using the grim trigger strategy is now defined by:

$$\widetilde{p} \equiv \max\left\{p \in \left[p^N, p^M\right] : \left(\frac{1}{1-\delta_n}\right)\pi\left(p, ..., p\right) \geq \max_{p_n < p} \pi\left(p_n, p, ..., p\right) + \left(\frac{\delta_n}{1-\delta_n}\right)\pi^N\right\};$$

[35] As Theorem 4 describes the central result of the paper, I have included the condition defining p^* in the statement of the theorem to make it more self-contained.

where firm n's incentive compatibility constraint will be the first to bind.[36]

From the "price matching" objective function for firm i of

$$W_i\left(p_i, \mathbf{p}_{-i}\right) \equiv \pi\left(p_i, \mathbf{p}_{-i}\right)+\left(\frac{\delta_i}{1-\delta_i}\right) \pi\left(p_i, \ldots, p_i\right),$$

we can define the best reply function,

$$\phi_i\left(\mathbf{p}_{-i}\right)=\max_{p_i} W_i\left(p_i, \mathbf{p}_{-i}\right),$$

and p_i^* as the fixed point. In contrast to the case of identical firms, p_i^* is now firm-specific as it depends on a firm's discount factor.

To characterize the relationship between a firm's discount factor and p_i^*, first note that

$$\frac{\partial^2 W_i(p, \ldots, p)}{\partial p_i \partial \delta}=\left(\frac{1}{\left(1-\delta_i\right)^2}\right) \sum_{j=1}^n \frac{\partial \pi(p, \ldots, p)}{\partial p_j}>0, \text{ if } p<p^M. \tag{7}$$

p_{i+1}^* is defined by:

$$\frac{\partial W_{i+1}\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_{i+1}}=\frac{\partial \pi\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_{i+1}}+\left(\frac{\delta_{i+1}}{1-\delta_{i+1}}\right) \sum_{j=1}^n \frac{\partial \pi\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_j}=0. \tag{8}$$

Substitute δ_i for δ_{i+1} in (8) and then using $\delta_i \geq \delta_{i+1}$ and (7), it follows that

$$\frac{\partial W_i\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_i}=\frac{\partial \pi\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_i}+\left(\frac{\delta_i}{1-\delta_i}\right) \sum_{j=1}^n \frac{\partial \pi\left(p_{i+1}^*, \ldots, p_{i+1}^*\right)}{\partial p_j} \geq 0.$$

The concavity of W_i then implies $p_i^* \geq p_{i+1}^*$. Hence,

$$p^N<p_n^* \leq p_{n-1}^* \leq \cdots \leq p_1^*<p^M.$$

Thus, more patient firms are willing to raise price to a higher level when acting as a price leader.

Theorem 5 *Allow firms to have heterogeneous discount factors. If A1-A3 and* $\left(p_1^0, \ldots, p_n^0\right)=\left(p^N, \ldots, p^N\right)$ *then* $\left\{\left(p_1^t, \ldots, p_n^t\right)\right\}_{t=1}^{\infty}$ *is weakly increasing over time and there exists finite* T *such that* $p_1^t=\cdots=p_n^t=\widehat{p}\ \forall t \geq T$ *where*

$$\widehat{p}\left\{\begin{array}{ll}\in\left\{p_1^*-\varepsilon, p_1^*, p_1^*+\varepsilon\right\} & \text{if } p_1^*+\varepsilon \leq \widetilde{p} \\ \in\left\{p_1^*-\varepsilon, p_1^*\right\} & \text{if } \widetilde{p}=p_1^* \\ =\widetilde{p} & \text{if } \widetilde{p} \leq p_1^*-\varepsilon\end{array}\right. .$$

[36] Firms are not allowed to coordinate on a collusive outcome with unequal market shares which is one way to improve collusion when firms have different discount factors; see Harrington (1989) and Obara and Zincenko (2011). This restriction would seem reasonable given that firms are tacitly colluding in which case it isn't clear how they would achieve mutual understanding regarding a market allocation without engaging in express communication.

Proof. Available on request.[37] ■

Firm i is sure to lead a price increase when: i) the current price is below p_i^*; ii) it doesn't expect other firms to lead in price; and iii) it expects other firms would at least match that price increase. As long as the resulting price does not exceed $\widetilde{p}$, other firms will indeed at least match the price increase. Thus, if $p_i^* \leq \widetilde{p}$ then, by A1-A3, price will eventually reach p_i^*. If $p_1^* \leq \widetilde{p}$ then price will climb all the way to p_1^*. The case of $p_1^* \leq \widetilde{p}$ occurs when firms are not too asymmetric in their discount factors.[38] In this situation, the constraint on the steady-state price is that no firm wants to be a price leader once price reaches p_1^*. Thus, Theorem 4 is unaffected if firms' discount factors are not too disparate. When instead firms are sufficiently different - so that $p_n^* < \widetilde{p} < p_1^*$ - the constraint on the steady-state price is instead that prices higher than $\widetilde{p}$ are not sustainable. While the more patient firms - such as firm 1 - would be willing to raise price beyond $\widetilde{p}$ if it was to be subsequently matched, the more impatient firms - such as firm n - would prefer to undercut such a price and, as a result, no firm raises price beyond $\widetilde{p}$.

8 Linear Example: Explicit vs. Tacit Collusion

The preceding analysis has shown that tacit collusion through price leadership and price matching results in a steady-state price of p^*. To make a comparison with explicit collusion, let us assume that firms, if they could expressly communicate, would agree to simultaneously raise price to the best equilibrium price of $\widetilde{p}$. The steady-state price differential between explicit and tacit collusion is then measured by $\widetilde{p}-p^*$. This measure does implicitly assume that the same punishment is deployed with explicit collusion as with tacit collusion which is likely not to be the case since presumably more punishments are available to firms if they can coordinate through express communication.[39] It is then best to think of $\widetilde{p}-p^*$ as isolating the effect of the method of coordination - price leadership versus express communication - while controlling for the mechanism that sustains the collusive outcome.

Assuming linear demand and cost functions, a firm's profit function is

$$\pi\left(p_i, \mathbf{p}_{-i}\right) = \left(a - bp_i + d\left(\frac{1}{n-1}\right)\sum_{j\neq i} p_j\right)\left(p_i - c\right), \text{where } a > bc > 0, b > d > 0.$$

The non-collusive stage game Nash equilibrium price and the joint profit-maximizing price are, respectively,

$$p^N = \frac{a+bc}{2b-d},\ p^M = \frac{a+(b-d)\,c}{2\,(b-d)}.$$

[37] The proof is very similar to the proofs of Theorems 2 and 3.

[38] Theorem 3 implies $p_n^* < \widetilde{p}$, in which case if $p_1^* \simeq p_n^*$ then $p_1^* < \widetilde{p}$.

[39] Though there is also the argument that express communication allows for re-negotiation which can weaken punishments; see McCutcheon (1997).

The price matching best reply function is

$$\phi\left(\mathbf{p}_{-i}\right)=\frac{a+(b-\delta d)\,c}{2\,(b-\delta d)}+\left(\frac{(1-\delta)\,d}{2\,(b-\delta d)}\right)\left(\frac{1}{n-1}\right)\sum_{j\neq i}p_j,$$

from which we can derive p^*:

$$p^*=\phi\left(p^*,...,p^*\right)\Rightarrow p^*=\frac{a+(b-\delta d)\,c}{2\,(b-\delta d)}+\left(\frac{(1-\delta)\,d}{2\,(b-\delta d)}\right)p^*\Rightarrow$$

$$p^*=\frac{a+(b-\delta d)\,c}{2b-(1+\delta)\,d}$$

p^* is an increasing convex function of the discount factor:

$$\frac{\partial p^*}{\partial\delta}=\frac{d\,(a-(b-d)\,c)}{(2b-(1+\delta)\,d)^2}>0,\ \frac{\partial^2 p^*}{\partial\delta^2}=\frac{d^2\,(a-(b-d)\,c)}{(2b-(1+\delta)\,d)^3}>0.$$

It is straightforward to derive price under explicit collusion by solving (2):

$$\begin{aligned}\widetilde{p} \;&=\; \min\left\{\frac{4ab^2+ad^2+4b^3c+bcd^2-4b^2cd-ad^2\delta-4abd+4abd\delta+3bcd^2\delta-4b^2cd\delta}{6bd^2-12b^2d+d^3\delta+8b^3-d^3-2bd^2\delta},\right.\\ &\left.\frac{a+(b-d)\,c}{2\,(b-d)}\right\}.\end{aligned}$$

If $\widetilde{p}<p^M$ then $\widetilde{p}$ is also an increasing convex function of the discount factor:

$$\frac{\partial\widetilde{p}}{\partial\delta}=\frac{4bd\,(a-bc+cd)\,(2b-d)}{(4b\,(b-d)+d^2\,(1-\delta))^2}>0,\ \frac{\partial^2\widetilde{p}}{\partial\delta^2}=\frac{8bd^3\,(a-(b-d)\,c)\,(2b-d)}{(4b\,(b-d)+d^2\,(1-\delta))^3}>0.$$

Define $\delta^*\in(0,1)$ by: if $\delta<(>)\,\delta^*$ then $\widetilde{p}\,(\delta)<(=)\,p^M$.

The next result shows that $\widetilde{p}-p^*$ is increasing in δ when δ is low - so that a higher discount factor exacerbates the cost from coordinating through price leadership - but is decreasing in δ when δ is high.

Theorem 6 *Assume linear demand and cost functions. Then* $\frac{\partial(\widetilde{p}-p^*)}{\partial\delta}>(<)\,0$ *as* $\delta<(>)\,\delta^*$.

Illustrating this result for $a=1,b=1,d=.9,c=0$, Figures 1 and 2 compare price under explicit collusion and tacit collusion, and how this comparison varies with the discount factor. To begin, the forces determining the steady-state price depends on the method of coordination. With tacit collusion and price leadership, a firm that leads on price trades off lower current profit - as its demand falls by raising its price - and higher future profit - as rivals subsequently match that price. With a current loss and a future gain, a firm is more willing to engage in price leadership when its discount factor is higher; hence p^* is increasing in δ. It is then the profitability of leading that determines the steady-state price under tacit collusion. By comparison, explicit collusion allows firms to simultaneously raise price so there is no price leader and thus no current loss incurred; what constrains the collusive price is sustainability and, by the usual argument, $\widetilde{p}$ is increasing in δ (when $\widetilde{p} < p^M$).[40] In sum, the steady-state price under explicit collusion is determined by the profitability of not undercutting that price, while the profitability of leading a price increase is what drives the steady-state price under tacit collusion.

When the discount factor is low, price under explicit collusion is near the competitive price because only prices close to the competitive price are sustainable. Price under tacit collusion is also near the competitive price because only for small price increases above the competitive price is the current loss exceeded by the future gain, and that is because the first-order current loss is zero when all firms price at p^N. Hence, when the discount factor is low, the type of coordination mechanism makes little difference. When the discount factor is high, the collusive price is near the joint profit maximum under either explicit or tacit collusion. Given firms' long-run view, high prices are sustainable and firms are strongly inclined to lead price increases. It is when the discount factor is moderate that the coordination mechanism makes the biggest difference. Firms are able to sustain high prices but no firm is willing to act as a price leader to get price to that level. For the numerical example in Figure 1 with $\delta = .7$, the competitive price is .91 and explicit collusion results in a price of 4.47 which is close to the joint profit maximum of 5.00; however, tacit collusion with price leadership results in a price of only 2.13. It is when firms are moderately patient that the means of coordination has the most significant impact on the steady-state price.

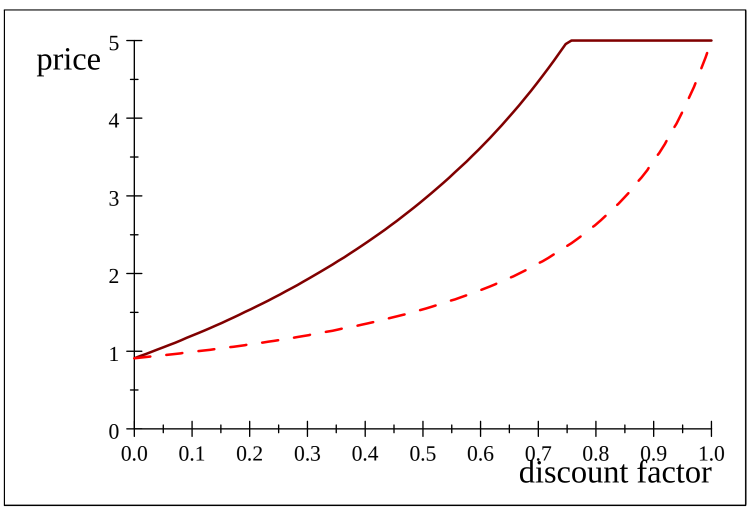

Figure 1: Price under explicit collusion (solid line) and tacit collusion (dashed line)

[40] Of course, sustainability is also an issue with tacit collusion. However, since $\widetilde{p} > p^*$ then incentive compatibility constraints are not binding under tacit collusion (at least when firms are not too asymmetric).

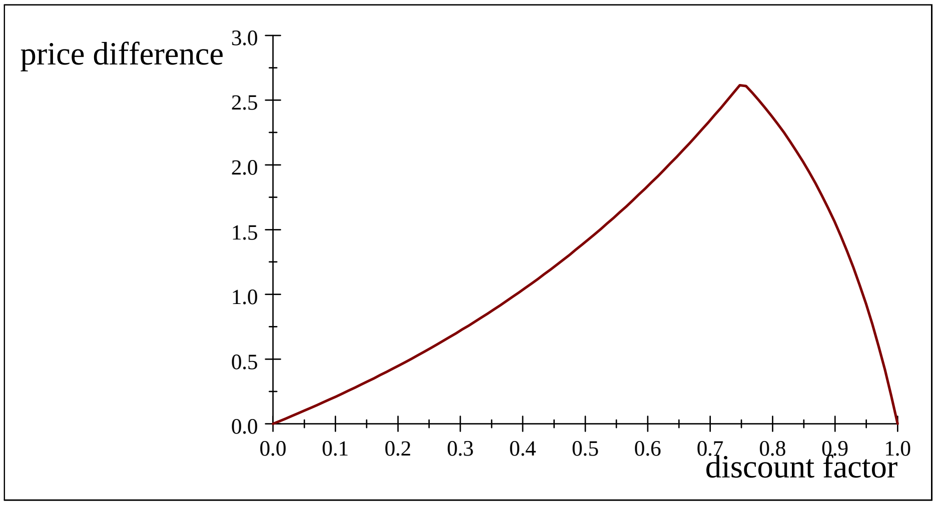

Figure 2: Price difference between explicit and tacit collusion, $\widetilde{p} - p^*$

This insight may also have implications for when cartel formation (that is, explicit collusion) is most likely. When the discount factor is sufficiently low, cartel formation is unlikely because the rise in price is small (whether firms, in the absence of cartel formation, would compete or tacitly collude). When the discount factor is sufficiently high, cartel formation is not likely either if the alternative is tacit collusion because tacit collusion does nearly as well.[41] It is when the discount factor is moderate that cartel formation is most attractive because it results in a much higher price than if firms either competed or tacitly colluded. While the attractiveness of tacit collusion (compared to competition) is always greater when the discount factor is higher, that is not the case with the attractiveness of explicit collusion (compared to tacit collusion). What are conditions promoting collusion can then depend on whether collusion is explicit or tacit.

9 Concluding Remarks

In his classic examination of imperfect competition, Chamberlain (1948) originally argued that collusion would naturally emerge because each firm would recognize the incentive to maintain a collusive price, rather than undercut its rivals' prices and bring forth retaliation. We now know that it is a non-trivial matter for firms to coordinate on a collusive solution because there are so many collusive equilibria. These equilibria differ in terms of the mechanism that sustains collusion as well as the particular outcome that is sustained. Modern oligopoly theory has generally ignored the question of how a collusive arrangement is achieved and instead focused on what can

[41] This results is at best suggestive because it comes with at least two serious caveats. First, if more severe punishments can be coordinated upon under explicit collusion then price will be higher. Second, the comparison focuses on steady-state profit and ignores how the transition path might differ between tacit and explicit collusion.

be sustained; that is, the properties of equilibrium outcomes. While the mutual understanding implicit in equilibrium can be acquired through express communication, this leaves unaddressed non-explicit forms of collusion, which are accepted by economists and the courts to occur in practice and are well-documented by experimental evidence.[42] This lack of theoretical attention to the distinction between explicit and tacit collusion has prevented advances in our understanding of how the means of coordination impacts the form and extent of collusion and, as a consequence, limited the role of economic theory in defining the contours of what is legal and illegal according to antitrust law.

The primary contribution of this paper is to characterize what collusive pricing looks like when firms deploy tacit means of coordination, specifically, price leadership. A model of tacit collusion requires jettisoning the assumption of equilibrium and instead imposing plausible assumptions on what firms commonly believe about their behavior. With mutual understanding about the method of tacit collusion - price leadership with price increases that are at least matched - but not about who leads and at what price, it proved possible to characterize the steady-state price. If firms are not too asymmetric, the steady-state price under tacit collusion is strictly less than the maximal equilibrium price and, therefore, less than the price that could be achieved with explicit collusion. While tacit coordination avoids the possibility of legal action, it produces a lower price than if firms were to expressly communicate. Thus, if the threat of penalties due to antitrust enforcement deters firms from engaging in explicit collusion, there is a welfare gain even if firms manage to tacitly collude.

The importance of understanding the distinction between explicit and tacit collusion is especially acute when it comes to policy. If the objective is to detect and prosecute cartels then explicit collusion is relevant in which case we need theories of explicit collusion to produce patterns to look for in the data. If the objective is to prevent horizontal mergers with coordinated effects then tacit collusion is most relevant in which case we need to know for what market structures tacit collusion is more likely to occur and lead to significant price increases. It is hoped that the progress that has been made here in developing a theory of tacit collusion will spur more research on modelling the distinction between explicit and tacit collusion, and thereby serve to close the gap between theory and practice on the matter of collusion.

[42] Some recent work showing the emergence of tacit collusion in an experimental setting includes Fonseca and Normann (2011) - who investigate when express means of coordination are especially valuable relative to tacit means - and Rojas (2011) - who shows that tacit collusion in the lab can be quite sophisticated in that the degree of collusion can vary with the current state of demand. For general references on tacit collusion in experiments, see Huck, Normann, and Oechssler (2004) and Engel (2007).

10 Appendix A

For when the price set is Δ_ε and $p^* \in \Delta_\varepsilon$, let us derive sufficient conditions for the property in (4) to hold, which is reproduced here:

$$\overline{\phi}(\mathbf{p}_{-i}) \begin{cases} \subseteq \{p' + \varepsilon, ..., p_i^*\} & \text{if } \mathbf{p}_{-i} \leq (p', ..., p') \text{ where } p' < p^* - \varepsilon \\ = \{p^*\} & \text{if } \mathbf{p}_{-i} = (p^*, ..., p^*) \\ \subseteq \{p^*, ..., p' - \varepsilon\} & \text{if } \mathbf{p}_{-i} \geq (p', ..., p') \text{ where } p' > p^* + \varepsilon \end{cases}$$

To show that this holds for $p' < p^* - \varepsilon$, it is sufficient to establish that a lower bound on $\overline{\phi}(p^* - \eta\varepsilon, ..., p^* - \eta\varepsilon)$ is $p^* - \eta\varepsilon + \varepsilon$ when $\eta \in \{2, 3, ...\}$. If the unconstrained optimum is at least $p^* - \eta\varepsilon + \varepsilon$ then that is indeed the case.

Define $\widehat{\phi}(p) \equiv \phi(p, ..., p)$ as the best reply function when all other firms price at p, and $\widehat{\phi} : \Re_+ \to \Re_+$. We want to show: if $\eta \in \{2, 3, ...\}$ then $\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon$. It will be shown that a sufficient condition for this result is $\widehat{\phi}'(p) \leq 1/2$. Note that:

$$\widehat{\phi}'(p) = -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2} + \left(\frac{\delta_i}{1-\delta_i}\right)\left(\frac{d^2 \pi}{dp^2}\right)} \leq -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2}},$$

so $\widehat{\phi}'(p) \leq 1/2$ holds when

$$-\frac{\partial^2 \pi}{\partial p_i^2} \geq 2\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}.$$

Using the functional forms in Section 8, $\widehat{\phi}'(p) < 1/2$ holds for the case of linear demand and cost:

$$\widehat{\phi}'(p) = \frac{(1-\delta)d}{2(b - \delta d)} < \frac{d}{2b} < \frac{1}{2}, \ \forall \delta \in (0, 1).$$

First note:

$$\widehat{\phi}(p^*) = p^*$$

and

$$\widehat{\phi}(p^*) = \widehat{\phi}(p^* - \varepsilon) + \int_{p^*-\varepsilon}^{p^*} \widehat{\phi}'(p)\, dp.$$

Given $\widehat{\phi}' \leq 1/2$, it follows from the previous equality:

$$\begin{aligned} \widehat{\phi}(p^*) &\leq \widehat{\phi}(p^* - \varepsilon) + \frac{\varepsilon}{2} \\ \widehat{\phi}(p^* - \varepsilon) &\geq \widehat{\phi}(p^*) - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - \varepsilon) \geq p^* - \frac{\varepsilon}{2} \end{aligned}$$

Next consider:

$$\begin{aligned} \widehat{\phi}(p^* - \varepsilon) &= \widehat{\phi}(p^* - 2\varepsilon) + \int_{p^*-2\varepsilon}^{p^*-\varepsilon} \widehat{\phi}'(p)\, dp \\ \widehat{\phi}(p^* - \varepsilon) &\leq \widehat{\phi}(p^* - 2\varepsilon) + \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - 2\varepsilon) \geq \widehat{\phi}(p^* - \varepsilon) - \frac{\varepsilon}{2} \end{aligned}$$

Using $\widehat{\phi}(p^* - \varepsilon) \geq p^* - \frac{\varepsilon}{2}$, the previous inequality implies:

$$\widehat{\phi}(p^* - 2\varepsilon) \geq p^* - \frac{\varepsilon}{2} - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - 2\varepsilon) \geq p^* - \varepsilon$$

which is the desired result for the case of $\eta = 2$. The proof is completed by induction.

Suppose for $\eta \geq 2$, it is true that:

$$\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon.$$

Consider:

$$\begin{aligned}
\widehat{\phi}(p^* - \eta\varepsilon) &= \widehat{\phi}(p^* - (\eta + 1)\varepsilon) + \int_{p^*-(\eta+1)\varepsilon}^{p^*-\eta\varepsilon} \widehat{\phi}'(p)\, dp \\
\widehat{\phi}(p^* - \eta\varepsilon) &\leq \widehat{\phi}(p^* - (\eta + 1)\varepsilon) + \frac{\varepsilon}{2} \\
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq \widehat{\phi}(p^* - \eta\varepsilon) - \frac{\varepsilon}{2}
\end{aligned}$$

Using $\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon$ in the preceding inequality,

$$\begin{aligned}
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq p^* - (\eta - 1)\varepsilon - \frac{\varepsilon}{2} \\
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq p^* - \eta\varepsilon + \frac{\varepsilon}{2} > p^* - \eta\varepsilon,
\end{aligned}$$

which proves the result. The proof when $p' > p^* + \varepsilon$ is analogous.

11 Appendix B

A useful property of other firms using PMP-compatible strategies is that a lower bound on a rational firm's period t continuation payoff is the payoff associated with all firms pricing at $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$ in all periods (Lemma 7). Intuitively, if the rivals to firm i are using PMP-compatible strategies then they will price at least as high as $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$ in all ensuing periods, as long as firm i does not violate the PMP property and induce a shift to p^N. Hence, firm i can at least earn the profit from all firms (including i) pricing at $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$.

Lemma 7 *Let V^t denote a firm's continuation payoff for period t. If the other firms' strategies are PMP-compatible and*

$$p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\} \ \ \forall j, \forall \tau \leq t - 1.$$

then, for a rational firm,

$$V^t \geq \frac{\pi\left(\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}, ..., \min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}\right)}{1 - \delta}.$$

Proof of Lemma 7. Wlog, the analysis will be conducted from the perspective of period 1 (and suppose 0 was a period of collusion). Consider firm i pricing at $\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}$ in the current period and then, in all ensuing periods, matching the maximum price of the other firms' in the previous period:

$$p_i^1=\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\};\ p_i^t=\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\ \text{ for } t=2,...$$

where

$$\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\equiv\max\left\{p_1^{t-1},...,p_{i-1}^{t-1},p_{i+1}^{t-1},...,p_n^{t-1}\right\}.$$

Given this strategy for firm i and that the other firms' strategies are PMP-compatible, there will never be a violation of the PMP property. Hence, firm i's payoff is

$$\pi\left(p_i^1,\mathbf{p}_{-i}^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},\mathbf{p}_{-i}^t\right).$$

Since $p_i^1\leq\max\left\{\mathbf{p}_{-i}^{t-1}\right\}$ (as all firms' strategies satisfy the PMP property) and $\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\leq p_j^t$ $\forall j\neq i$, $\forall t\geq 2$, it follows from firm i's profit being increasing in the other firms' prices that

$$\begin{aligned}&\pi\left(p_i^1,\mathbf{p}_{-i}^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},\mathbf{p}_{-i}^t\right)\\ \geq\ &\pi\left(p_i^1,...,p_i^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},...,\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right).\end{aligned}\tag{9}$$

Next note that $p_i^1\leq\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\leq\widetilde{p}\leq p^M$ which implies

$$\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},...,\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right)\geq\pi\left(p_i^1,...,p_i^1\right).$$

Using this fact on the RHS of (9),

$$\begin{aligned}&\pi\left(p_i^1,...,p_i^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},...,\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right)\\ \geq\ &\pi\left(p_i^1,...,p_i^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(p_i^1,...,p_i^1\right)=\frac{\pi\left(p_i^1,...,p_i^1\right)}{1-\delta_i}.\end{aligned}\tag{10}$$

(9) and (10) imply

$$\pi\left(p_i^1,\mathbf{p}_{-i}^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},\mathbf{p}_{-i}^t\right)\geq\frac{\pi\left(p_i^1,...,p_i^1\right)}{1-\delta_i},$$

from which we conclude $V^t\geq\pi\left(p_i^1,...,p_i^1\right)/\left(1-\delta\right)$. ■

Proof of Lemma 1. Suppose the period t history is such that

$$p_j^\tau < \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\} \text{ for some } j \text{ and some } \tau \leq t-1.$$

A PMP-compatible strategy has a firm price at p^N in the current and all future periods. Thus, if firm i's beliefs over other firms' strategies has support in S^{PMP} then pricing at p^N is clearly optimal. Hence, a PMP-compatible strategy is uniquely optimal for firm i for those histories.

For the remainder of the proof, suppose $p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\}$ $\forall j, \forall \tau \leq t-1$. To prove this lemma, we'll show that, for any strategy for firm i that does not satisfy the PMP property, there is a PMP-compatible strategy that yields a strictly higher payoff. Thus, regardless of firm i's beliefs over the other firms' strategies (as long as they have support in S^{PMP}), its expected payoff is strictly higher with some PMP-compatible strategy than with any PMP-incompatible strategy.

Given $p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\}$ $\forall j, \forall \tau \leq t-1$, firm i's strategy can violate the PMP property either by pricing above $\widetilde{p}$ or below $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$. Let us begin by considering a PMP-incompatible strategy that has firm i price at $p' > \widetilde{p}$. When its rivals price at $\mathbf{p}_{-i}^t$, a PMP-compatible strategy that has firm i price at $\widetilde{p}$ is more profitable than pricing at p' iff:

$$\pi\left(\widetilde{p}, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}, ..., \widetilde{p}\right) > \pi\left(p', \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}, ..., \widetilde{p}\right), \qquad (11)$$

where recall that the other firms will only follow price as high as $\widetilde{p}$. (11) holds iff

$$\pi\left(\widetilde{p}, \mathbf{p}_{-i}^t\right) > \pi\left(p', \mathbf{p}_{-i}^t\right). \qquad (12)$$

Given that $\mathbf{p}_{-i}^t \leq (\widetilde{p}, ..., \widetilde{p})$ and $p^N < \widetilde{p}$ then $\psi\left(\mathbf{p}_{-i}^t\right) \leq \psi\left(\widetilde{p}, ..., \widetilde{p}\right) < \widetilde{p}$. By the strict concavity of π in own price and that $\psi\left(\mathbf{p}_{-i}^t\right) < \widetilde{p} < p'$, (12) is true.

Next consider a PMP-incompatible strategy that has firm i price at $p'' < \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$. Let us show that a PMP-compatible strategy that has firm i price at $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ is more profitable than pricing at p'' for any $\mathbf{p}_{-i}^t \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$.[43] A sufficient condition for the preceding claim to be true is:

$$\begin{aligned} & \pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \mathbf{p}_{-i}^t\right) \\ & + \left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{\mathbf{p}_{-i}^t\right\}, ..., \max\left\{\mathbf{p}_{-i}^t\right\}\right) \\ > \; & \pi\left(p'', \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi^N, \end{aligned} \qquad (13)$$

where the LHS of (13) is a lower bound on the payoff from pricing at $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ and the RHS is the payoff from pricing at p''. In examining the LHS, note that

[43] Actually, it is shown to be only weakly as profitable when $\mathbf{p}_{-i}^t = (\widetilde{p}, ..., \widetilde{p})$.

$p_i^t = \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ and that the other firms' strategies are PMP-compatible imply

$$\max\left\{p_1^t, ..., p_n^t\right\} = \max\left\{\mathbf{p}_{-i}^t\right\}.$$

Using Lemma 7,

$$\left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{\mathbf{p}_{-i}^t\right\}, ..., \max\left\{\mathbf{p}_{-i}^t\right\}\right)$$

is a lower bound on the future payoff, which gives us the LHS of (13). When

$$\mathbf{p}_{-i}^t = \left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right), \tag{14}$$

(13) is

$$\begin{aligned}
&\pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) \\
&+\left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) \\
> \;&\pi\left(p'', \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) + \left(\frac{\delta}{1-\delta}\right)\pi^N.
\end{aligned} \tag{15}$$

Since $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} \leq \widetilde{p}$ then (15) is true for all $p'' < \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ as it is the equilibrium condition for a grim trigger strategy with collusive price $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$.[44] Thus, (13) holds for (14).

To complete the proof, it will be shown that the LHS of (13) is increasing in $\mathbf{p}_{-i}^t$ at a faster rate than the RHS in which case (13) holds for all

$$\mathbf{p}_{-i}^t \geq \left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right).$$

The derivative with respect to p_j^t, $j \neq i$, of the LHS of (13) is

$$\frac{\partial\pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \mathbf{p}_{-i}^t\right)}{\partial p_j} + \left(\frac{\delta}{1-\delta}\right)\frac{\partial \max\left\{\mathbf{p}_{-i}^t\right\}}{\partial p_j}\sum_{k=1}^{n}\frac{\partial\pi\left(\max\left\{\mathbf{p}_{-i}^t\right\}, ..., \max\left\{\mathbf{p}_{-i}^t\right\}\right)}{\partial p_k}, \tag{16}$$

and of the RHS of (13) is

$$\frac{\partial\pi\left(p'', \mathbf{p}_{-i}^t\right)}{\partial p_j}. \tag{17}$$

(16) exceeds (17) because the second term in (16) is non-negative, given that $\mathbf{p}_{-i}^t \leq \left(p^M, ..., p^M\right)$, and the first term of (16) exceeds (17) because $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} > p''$ and

$$\frac{\partial^2\pi}{\partial p_i \partial p_j} > 0, \; j \neq i.$$

[44] Recall that $\widetilde{p}$ is the highest price consistent with the grim trigger strategy being an equilibrium. Note that (15) holds with equality when $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} = \widetilde{p}$ and $p'' = \psi\left(\widetilde{p}, ..., \widetilde{p}\right)$ and otherwise is a strict inequality.

■

Proof of Theorem 2. Given firms' strategies are PMP-compatible then it immediately follows from $(p_1^0, ..., p_n^0) = (p^N, ..., p^N)$ that each firm's price is weakly increasing. Given a finite price set and the boundedness and monotonicity of prices, prices converge in finite time. Hence, there exists $\widehat{p} \in \{p^N, ..., \widetilde{p}\}$ and finite T such that $p_1^t = \cdots = p_n^t = \widehat{p}$ for all $t \geq T$.

The remainder of the proof entails proving $\widehat{p} \leq p^* + \varepsilon$. If $p^* + \varepsilon \geq \widetilde{p}$ then, given that PMP-compatible strategies do not have firms pricing above $\widetilde{p}$, it is immediate that $\widehat{p} \leq p^* + \varepsilon$. Thus, suppose $p^* + \varepsilon < \widetilde{p}$. Before going any further, an overview of the proof is provided. First it is shown that if a firm is rational and believes the other firms use PMP-compatible strategies then a firm will not price at $\widetilde{p}$. The reason is that firm i would find it optimal to price above $p^* + \varepsilon$ only if it induced at least one of its rivals to enact further price increases (and not just match the firm's price). However, if a firm believes its rivals will not price above $\widetilde{p}$ (which follows from believing its rivals use PMP-compatible strategies) then it is not optimal for a firm to raise price to $\widetilde{p}$ because it can only expect its rivals to match a price of $\widetilde{p}$, not exceed it. This argument works as well to show that each of the other firms will not raise price to $\widetilde{p}$. Hence, there is an upper bound on price of $\widetilde{p} - \varepsilon$. The proof is completed by induction using the common knowledge in A1-A2. If a firm believes its rivals will not price above p' then it can be shown that a firm will find it optimal not to price above $p' - \varepsilon$. This argument works only when $p' \geq p^* + 2\varepsilon$ which implies that an upper bound on price is $p^* + \varepsilon$, which is the desired result.

Define $\overline{\phi}^U(\mathbf{p}_{-i})$ to be the maximal element of $\overline{\phi}(\mathbf{p}_{-i})$ and let us show that $\overline{\phi}^U(\mathbf{p}_{-i})$ is non-decreasing in $\mathbf{p}_{-i}$. By the definition of $\overline{\phi}^U(\mathbf{p}_{-i})$, we know that:

$$W\left(\overline{\phi}^U(\mathbf{p}'_{-i}), \mathbf{p}'_{-i}\right) - W(p_i, \mathbf{p}'_{-i}) > 0, \ \forall p_i \in A \equiv \left\{p \in \Delta_\varepsilon : p > \overline{\phi}^U(\mathbf{p}'_{-i})\right\}.$$

Since $\frac{\partial^2 W(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} = \frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} > 0$ then $p_i > \overline{\phi}^U(\mathbf{p}'_{-i})$ and $\mathbf{p}''_{-i} \leq \mathbf{p}'_{-i}$ imply

$$W(p_i, \mathbf{p}'_{-i}) - W(p_i, \mathbf{p}''_{-i}) \geq W\left(\overline{\phi}^U(\mathbf{p}'_{-i}), \mathbf{p}'_{-i}\right) - W\left(\overline{\phi}^U(\mathbf{p}'_{-i}), \mathbf{p}''_{-i}\right), \ \forall p_i \in A,$$

and, re-arranging, we have

$$W\left(\phi^U(\mathbf{p}'_{-i}), \mathbf{p}''_{-i}\right) - W(p_i, \mathbf{p}''_{-i}) \geq W\left(\overline{\phi}^U(\mathbf{p}'_{-i}), \mathbf{p}'_{-i}\right) - W(p_i, \mathbf{p}'_{-i}), \ \forall p_i \in A.$$

Therefore,

$$W\left(\overline{\phi}^U(\mathbf{p}'_{-i}), \mathbf{p}''_{-i}\right) - W(p_i, \mathbf{p}''_{-i}) > 0, \ \forall p_i \in A,$$

which, along with $\mathbf{p}''_{-i} \leq \mathbf{p}'_{-i}$ and the strict concavity of W in own price, imply $\overline{\phi}^U(\mathbf{p}''_{-i}) \leq \overline{\phi}^U(\mathbf{p}'_{-i})$. Hence, $\overline{\phi}^U(\mathbf{p}_{-i})$ is non-decreasing.

By (4), $p' > p^* + \varepsilon$ implies $\overline{\phi}^U(p', ..., p') \leq p' - \varepsilon$. Given $\overline{\phi}^U(\mathbf{p}_{-i})$ is non-decreasing in $\mathbf{p}_{-i}$, it follows:

$$\text{if } \mathbf{p}_{-i} \leq (p', ..., p') \text{ and } p' > p^* + \varepsilon \text{ then } \overline{\phi}^U(\mathbf{p}_{-i}) \leq p' - \varepsilon. \tag{18}$$

From the strict concavity of W in own price, we have:

$$\text{if } p'' > p' \geq \overline{\phi}^U(\mathbf{p}_{-i}) \text{ then} \tag{19}$$

$$\pi(p', \mathbf{p}_{-i}) + \left(\frac{\delta}{1-\delta}\right)\pi(p', ..., p') > \pi(p'', \mathbf{p}_{-i}) + \left(\frac{\delta}{1-\delta}\right)\pi(p'', ..., p'').$$

This property will be used in the ensuing proof.

Let us show that if firm i believes the other firms' strategies are PMP-compatible then a price of $\widetilde{p} - \varepsilon$ is strictly preferred to $\widetilde{p}$. Firm i's beliefs on $\mathbf{p}_{-i}^t$ have support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$. For any $\mathbf{p}_{-i}^t \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$, Lemma 7 implies that a lower bound on its payoff from $p_i^t = \widetilde{p} - \varepsilon$ is

$$\pi\left(\widetilde{p} - \varepsilon, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p} - \varepsilon, ..., \widetilde{p} - \varepsilon\right). \tag{20}$$

For any $\mathbf{p}_{-i}^t \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$, it follows from all firms using PMP-compatible strategies that firm i's payoff from $p_i^t = \widetilde{p}$ is

$$\pi\left(\widetilde{p}, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}, ..., \widetilde{p}\right). \tag{21}$$

Given that $\widetilde{p} > p^* + \varepsilon$ then $\overline{\phi}^U\left(\mathbf{p}_{-i}^t\right) \leq \widetilde{p} - \varepsilon$ for all $\mathbf{p}_{-i}^t \leq (\widetilde{p}, ..., \widetilde{p})$ by (18). It then follows from (19) that (20) strictly exceeds (21). Therefore, for any beliefs of firm i with support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$, a price of $\widetilde{p} - \varepsilon$ is strictly preferred to $\widetilde{p}$. It follows that if a firm is rational and believes the other firms use PMP-compatible strategies then its optimal price does not exceed $\widetilde{p} - \varepsilon$.

Given the common knowledge from Assumptions A1-A2, it is also the case that firm i believes firm j $(\neq i)$ is rational and that firm j believes firm h (for all $h \neq j$) uses a PMP-compatible strategy. Hence, applying the preceding argument to firm j, firm i believes firm j will not price above $\widetilde{p} - \varepsilon$. Firm i's beliefs on $\mathbf{p}_{-i}^t$ then have support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p} - \varepsilon\right]^{n-1}$. If $\widetilde{p} - \varepsilon > p^* + \varepsilon$ then, by (18), $\overline{\phi}^U\left(\mathbf{p}_{-i}^t\right) \leq \widetilde{p} - 2\varepsilon$ for all $\mathbf{p}_{-i}^t \leq (\widetilde{p} - \varepsilon, ..., \widetilde{p} - \varepsilon)$.[45] By the same logic as above, a lower bound on firm i's payoff from $p_i^t = \widetilde{p} - 2\varepsilon$ is

$$\pi\left(\widetilde{p} - 2\varepsilon, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p} - 2\varepsilon, ..., \widetilde{p} - 2\varepsilon\right), \tag{22}$$

[45] If instead $\widetilde{p} - \varepsilon \leq p^* + \varepsilon$ then, given that it has already been shown $\widetilde{p} - \varepsilon$ is an upper bound on the limit price, it follows that $p^* + \varepsilon$ is an upper bound and we're done.

while its payoff from $p_i^t = \widetilde{p} - \varepsilon$ is

$$\pi\left(\widetilde{p} - \varepsilon, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right) \pi\left(\widetilde{p} - \varepsilon, ..., \widetilde{p} - \varepsilon\right). \tag{23}$$

With (23), we used the fact that firms will not price above $\widetilde{p} - \varepsilon$, which was derived in the first step. Again using (19), it is concluded that (22) strictly exceeds (23). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^t$ with support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p} - \varepsilon\right]^{n-1}$, a price of $\widetilde{p} - 2\varepsilon$ is strictly preferred to $\widetilde{p} - \varepsilon$. It follows that if a firm is rational and a firm believes other firms' strategies are PMP-compatible, believes the other firms are rational, and believes each of the other firms believes its rivals use PMP-compatible strategies then a firm's optimal price does not exceed $\widetilde{p} - 2\varepsilon$. Hence, all firms will not price above $\widetilde{p} - 2\varepsilon$.

The proof is completed by induction. Suppose we have shown that firm i believes that the other firms will not price above p' so firm i's beliefs on $\mathbf{p}_{-i}^t$ have support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, p'\right]^{n-1}$. (That we can get to the point that firms have those beliefs relies on rationality and that firms use PMP-compatible strategies are both common knowledge.) If $p' > p^* + \varepsilon$ then $\overline{\phi}^U\left(\mathbf{p}_{-i}^t\right) \leq p' - \varepsilon$ for all $\mathbf{p}_{-i}^t \leq (p', ..., p')$. A lower bound on firm i's payoff from $p_i^t = p' - \varepsilon$ is

$$\pi\left(p' - \varepsilon, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right) \pi\left(p' - \varepsilon, ..., p' - \varepsilon\right), \tag{24}$$

while its payoff from $p_i^t = p'$ is

$$\pi\left(p', \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right) \pi\left(p', ..., p'\right), \tag{25}$$

since all firms have an upper bound of p' on their prices. Using (19), it is concluded that (24) strictly exceeds (25). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^t$ with support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, p'\right]^{n-1}$, a price of $p' - \varepsilon$ is strictly preferred to p'. It follows that firms' prices are bounded above by $p' - \varepsilon$. The preceding argument is correct as long as $p' > p^* + \varepsilon$; therefore, price is bounded above by $p^* + \varepsilon$. ■

Proof of Theorem 3. p^* is defined by

$$\frac{\partial W\left(p^*, ..., p^*\right)}{\partial p_i} = \frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right) \sum_{j=1}^{n} \frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_j} = 0$$

or

$$\frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_i} + \delta \sum_{j \neq i}^{n} \frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_j} = 0.$$

For all $p \geq p^M$,

$$\frac{\partial \pi\left(p, ..., p\right)}{\partial p_i} < 0 \text{ and } \sum_{j=1}^{n} \frac{\partial \pi\left(p, ..., p\right)}{\partial p_j} \leq 0,$$

which implies $p^* < p^M$ by the strict concavity of W in own price. To show $p^* > p^N$, note that $\phi(p,...,p) > \psi(p,...,p)$ and $\psi(p,...,p) \geq p$ $\forall p \leq p^N$ implies $\phi(p,...,p) > p$ $\forall p \leq p^N$. Since $\phi(p,...,p) \gtreqless p$ as $p \lesseqgtr p^*$ then $p^* > p^N$. We have then shown $p^* \in (p^N, p^M)$.

If $\widetilde{p} = p^M$ then $p^* \in (p^N, \widetilde{p})$ and we are done. From hereon, suppose $\widetilde{p} < p^M$ in which case the incentive compatibility constraint (ICC) binds:

$$\frac{\pi(\widetilde{p},...,\widetilde{p})}{1-\delta} = \pi(\psi(\widetilde{p},...,\widetilde{p}), \widetilde{p},...,\widetilde{p}) + \left(\frac{\delta}{1-\delta}\right)\pi(p^N,...,p^N) \; . \tag{26}$$

As $p \in (p^N, p^*]$ implies $\psi(p,...,p) < p \leq \phi(p,...,p)$ then, by the strict concavity of W,

$$W(p,...,p) > W(\psi(p), p,...,p),$$

which is equivalently expressed as

$$\frac{\pi(p,...,p)}{1-\delta} > \pi(\psi(p), p,...,p) + \left(\frac{\delta}{1-\delta}\right)\pi(\psi(p),...,\psi(p)). \tag{27}$$

$p > p^N$ implies $\psi(p,...,p) \in (p^N, p)$. Next note $\psi(p,...,p) < p \leq p^* < p^M$ implies $\psi(p,...,p) < p^M$. It then follows from $\psi(p,...,p) \in (p^N, p^M)$ that $\pi(\psi(p),...,\psi(p)) > \pi(p^N,...,p^N)$. Using this property in (27), we have

$$\frac{\pi(p,...,p)}{1-\delta} > \pi(\psi(p), p,...,p) + \left(\frac{\delta}{1-\delta}\right)\pi(p^N,...,p^N), \; \forall p \in (p^N, p^*]. \tag{28}$$

Therefore, $p \in (p^N, p^*]$ is sustainable with the grim trigger strategy. Given (26) - where the ICC binds for $p = \widetilde{p}$ - and evaluating (28) at $p = p^*$ - so the ICC does not bind - it follows from (2) that $\widetilde{p} > p^*$. ■

Proof of Theorem 4. Given that $\widehat{p} \leq p^* + \varepsilon$ by Theorem 2, it is only necessary to show $\widehat{p} \geq p^* - \varepsilon$. Thus, suppose $\widehat{p} < p^* - \varepsilon$ - that is, we are considering strategies consistent with A1-A2 for which there exists t' such that $p_j^t = \widehat{p}$ $\forall j, \forall t \geq t'$ - and let us derive a contradiction.

By A3, this means $\exists T$ such that, for $t > t' + T$, a firm's expected payoff from using a strategy that converges to $\widehat{p}$ is

$$\frac{\pi(\widehat{p},...,\widehat{p})}{1-\delta}. \tag{29}$$

By Lemma 7, a lower bound on firm 1's payoff from pricing instead at $\phi(\widehat{p},...,\widehat{p})$ is

$$\pi(\phi(\widehat{p},...,\widehat{p}), \widehat{p},...,\widehat{p}) + \left(\frac{\delta}{1-\delta}\right)\pi(\phi(\widehat{p},...,\widehat{p}),...,\phi(\widehat{p},...,\widehat{p})). \tag{30}$$

It follows from $\widehat{p} < p^* - \varepsilon$ that (30) strictly exceeds (29) and, therefore, this firm's strategy is not rational given its beliefs as to other firms' strategies. Hence, it must be the case that $\widehat{p} \geq p^* - \varepsilon$ which, when combined with Theorem 2, implies $\widehat{p} \in \{p^* - \varepsilon, p^*, p^* + \varepsilon\}$. ■

Proof of Theorem 6. Given that $\frac{\partial p^*}{\partial \delta} > 0$ and $\frac{\partial \widetilde{p}}{\partial \delta} = 0$ for $\delta > \delta^*$ then: if $\delta > \delta^*$ then $\frac{\partial(\widetilde{p} - p^*)}{\partial \delta} < 0$. The remainder of the proof focuses on showing: if $\delta < \delta^*$ then $\frac{\partial(\widetilde{p} - p^*)}{\partial \delta} > 0$.

If $\delta < \delta^*$ then

$$\begin{aligned}\widetilde{p} - p^* &= \frac{4ab^2 + ad^2 + 4b^3c + bcd^2 - 4b^2cd - ad^2\delta - 4abd + 4abd\delta + 3bcd^2\delta - 4b^2cd\delta}{6bd^2 - 12b^2d + d^3\delta + 8b^3 - d^3 - 2bd^2\delta} \\ &\quad - \frac{a + (b - \delta d)c}{2b - (1 + \delta)d}\end{aligned}$$

$$\frac{\partial(\widetilde{p} - p^*)}{\partial \delta} = \left[\frac{d(a - bc + dc)}{\left(8b^3 - 4b^2d\delta - 12b^2d + 2bd^2\delta + 6bd^2 + d^3\delta^2 - d^3\right)^2}\right] \times \Psi(\delta), \quad (31)$$

where

$$\begin{aligned}\Psi(\delta) \equiv\ & 16b^4 - 32b^3d\delta - 16b^3d + 8b^2d^2\delta^2 + 40b^2d^2\delta - \\ & 4bd^3\delta^2 - 16bd^3\delta + 4bd^3 - d^4\delta^2 + 2d^4\delta + d^4.\end{aligned}$$

As the term in [ ] in (31) is positive then

$$sign\left\{\frac{\partial(\widetilde{p} - p^*)}{\partial \delta}\right\} = sign\{\Psi(\delta)\}$$

In evaluating the sign of $\Psi(\delta)$, first note it is positive at the extreme values of δ:

$$\Psi(0) = 16b^4 - 16b^3d + 4bd^3 - d^4 = 16b^3(b - d) + d^3(4b - d) > 0$$

$$\begin{aligned}\Psi(1) &= 16b^4 - 48b^3d + 48b^2d^2 - 16bd^3 = 16b\left(b^3 - 3b^2d + 3bd^2 - d^3\right) \\ &= 16b\left(b^2(b - d) - 2bd(b - d) + d^2(b - d)\right) = 16b(b - d)\left(b^2 - 2bd + d^2\right) \\ &= 16b(b - d)(b - d)^2 > 0,\end{aligned}$$

which follow from $b > d > 0$. Given $\Psi(0), \Psi(1) > 0$, if $\Psi(\delta)$ is weakly monotonic then $\Psi(\delta) > 0 \forall \delta \in [0, 1]$ and thus $\Psi(\delta) > 0 \forall \delta \in [0, \delta^*)$. Let us show $\Psi'(\delta) < 0$. Consider:

$$\Psi'(\delta) = 40b^2d^2 - 32b^3d - 2d^4\delta - 16bd^3 + 2d^4 + 16b^2d^2\delta - 8bd^3\delta.$$

Since

$$\Psi''(\delta) = -2d^4 + 16b^2d^2 - 8bd^3 = 2d^2\left(8b^2 - 4bd - d^2\right) > 0,$$

$\Psi'(1) < 0$ is a sufficient condition to establish that $\Psi'(\delta) < 0 \forall \delta \in [0, 1]$. Given that

$$\begin{aligned}\Psi'(1) &= 56b^2d^2 - 32b^3d - 24bd^3 = 8bd\left(7bd - 4b^2 - 6d^2\right) \\ &= -8bd(b - d)(4b - 3d) < 0,\end{aligned}$$

we are done. ■

12 Appendix C

In deriving sufficient conditions for $\left(S^L, S^F\right)$ to be a subgame perfect equilibrium, let us first consider S^L have ρ denote the lagged maximum price. If $\rho = p^N$ then $S^L\left(p^N\right) = p'$ which is optimal iff p' is at least as profitable as p^N,

$$\begin{aligned} &\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \\ \geq\; &\pi\left(p^N, p^N\right) + \delta\pi\left(p', p^N\right) + \delta^2\pi\left(p^M, p'\right) + \left(\frac{\delta^3}{1-\delta}\right)\pi\left(p^M, p^M\right) \end{aligned} \tag{32}$$

and at least as profitable as p^M,

$$\begin{aligned} &\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \\ \geq\; &\pi\left(p^M, p^N\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right). \end{aligned} \tag{33}$$

(32) and (33) can be simplified to:

$$\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \delta^2\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p^N\right) + \delta\pi\left(p', p^N\right) + \delta^2\pi\left(p^M, p'\right) \tag{34}$$

$$\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) \geq \pi\left(p^M, p^N\right) + \delta\pi\left(p^M, p^M\right) \tag{35}$$

If $\delta \simeq 1$ then (34) is true, and (35) is true when:

$$\pi\left(p', p^N\right) + \pi\left(p^M, p'\right) > \pi\left(p^M, p^N\right) + \pi\left(p^M, p^M\right) \tag{36}$$

Now suppose $\rho = p'$. $S^L\left(p'\right) = p^M$ is optimal iff p^M is at least as profitable as p^N,

$$\pi\left(p^M, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{37}$$

and at least as profitable as p',

$$\begin{aligned} \pi\left(p^M, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \;\geq\; &\pi\left(p', p'\right) + \delta\pi\left(p^M, p'\right) \\ &+ \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right). \end{aligned} \tag{38}$$

If $\delta \simeq 1$ then (37) and (38) hold. Finally, if $\rho = p^M$ then $S^L\left(p^M\right) = p^M$ is optimal iff:

$$\left(\frac{1}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \max\left\{\pi\left(p^N, p^M\right), \pi\left(p', p^M\right)\right\} + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{39}$$

which holds if $\delta \simeq 1$. In sum, S^L is subgame perfect if $\delta \simeq 1$ and (36) holds.

Next, let us turn to S^F. If $\rho = p^N$ then $S^F\left(p^N\right) = p^N$ is optimal iff p^N is at least as profitable as p',

$$\pi\left(p^N, p'\right)+\delta\pi\left(p', p^M\right)+\left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p', p'\right)+\delta\pi\left(p', p^M\right)+\left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right), \tag{40}$$

and is at least as profitable as p^M,

$$\pi\left(p^N, p'\right)+\delta\pi\left(p', p^M\right)+\left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^M, p'\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right). \tag{41}$$

Both conditions hold for all δ.[46] If $\rho = p'$ then $S^F\left(p'\right) = p'$ is optimal iff p' is at least as profitable as p^N,

$$\pi\left(p', p^M\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p'\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{42}$$

and is at least as profitable as p^M,

$$\pi\left(p', p^M\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \left(\frac{1}{1-\delta}\right)\pi\left(p^M, p^M\right). \tag{43}$$

(42) holds for $\delta \simeq 1$, and (43) holds for all δ. Finally, if $\rho = p^M$ then $S^F\left(p^M\right) = p^M$ is optimal iff (39) is true. In sum, S^F is subgame perfect if $\delta \simeq 1$.

To evaluate when (36) holds, consider:

$$\begin{aligned}
\pi\left(p', p^N\right)+\pi\left(p^M, p'\right) &> \pi\left(p^M, p^N\right)+\pi\left(p^M, p^M\right) \Leftrightarrow \\
\pi\left(\frac{p^M+p^N}{2}, p^N\right)-\pi\left(p^M, p^N\right) &> \pi\left(p^M, p^M\right)-\pi\left(p^M, \frac{p^M+p^N}{2}\right) \Leftrightarrow \\
-\int_{\frac{p^M+p^N}{2}}^{p^M}\left(\frac{\partial\pi\left(p, p^N\right)}{\partial p_1}\right)dp_1 &> \int_{\frac{p^M+p^N}{2}}^{p^M}\left(\frac{\partial\pi\left(p^M, p\right)}{\partial p_2}\right)dp_2.
\end{aligned} \tag{44}$$

Assuming linear demand and constant marginal cost,

$$\pi\left(p_i, \mathbf{p}_{-i}\right) = \left(a - bp_i + d\left(\frac{1}{n-1}\right)\sum_{j\neq i} p_j\right)\left(p_i - c\right), \text{where } a > bc > 0, b > d > 0,$$

[46] Note that $\pi\left(p^N, p'\right) > \pi\left(p', p'\right)$ for if that was not the case then p' would be a static Nash equilibrium and thereby violation the assumption that p^N is the unique Nash equilibrium. Similarly, it must be true that $\pi\left(p', p^M\right) > \pi\left(p^M, p^M\right)$.

(44) is

$$-\int_{\frac{p^M+p^N}{2}}^{p^M}\left(a+bc-2bp_1+dp^N\right)dp_1 \quad > \quad \int_{\frac{p^M+p^N}{2}}^{p^M} d\left(p^M-c\right)dp_2 \Leftrightarrow$$

$$-\left(a+bc+dp^N\right)\left(\frac{p^M-p^N}{2}\right)+b\left[\left(p^M\right)^2-\left(\frac{p^M+p^N}{2}\right)^2\right] \quad > \quad d\left(p^M-c\right)\left(\frac{p^M-p^N}{2}\right)$$

which, after some manipulations, is equivalent to

$$3bp^M+bp^N > 2a+2bc+2dp^N+2dp^M-2dc. \tag{45}$$

Substituting

$$p^N=\frac{a+bc}{2b-d},\ p^M=\frac{a+(b-d)\,c}{2\,(b-d)}$$

and again performing some manipulations, (45) is equivalent to

$$[a+(b-d)\,c]\left[(6b-4d)\,(b-d)+d^2\right]+2\,(b-2d)\,(b-d)\,dc>0. \tag{46}$$

The first term is positive because $b>d$, while the second term is non-negative when $b\geq 2d$. Hence, if products are sufficiently differentiated then (46) is true. When instead $b<2d$ then (46) holds when $c\simeq 0$. Hence, if cost is sufficiently small then (46) is true.

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